Optimized Random Network Coding for Reliable Multicast Communications

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***Abstract*—In this letter, we propose two optimized multicast communication strategies based on the Network Coding principle which aim to significantly improve the performance in terms of the power cost and delivery delay associated to the transmission of the whole data flow. The proposed strategies are of special interest for service delivered in an unreliable mode. The reported numerical results clearly show that both the strategies achieve the aforementioned goals in comparison with the Random Linear Network Coding alternative.**

I. Introduction

Multicast and broadcast services will play an important role in the future broadband wireless networks area [1], [2]. With reference to this aspect, an important issue that has to be carefully considered is how to match the end-user Quality of Service (QoS) profiles.

Despite recent proposals [3], little attention has been paid to the problem of matching a QoS level on a Multicast Group (MG) basis, for example, the problem of delivering services with a target QoS profile to *all* the User Equipments (UEs) belonging to a same MG. In addition to this, it is also important to consider that due to the complexity of delivering multicast services in a reliable mode, the next 4G wireless networks (i.e., the 3GPP's LTE-Advanced) have not foreseen the use of any error control scheme. For this reason this letter considers a system model where UEs can not acknowledge any received data flow.

In such a context, a promising approach to reducing at the same time the drawbacks of Point-to-Multipoint (PtM) communications and match the implementation constraints seems to be the Network Coding (NC) principle [4], [5]. In particular, several works [6] have clearly shown the effectiveness of NC schemes in PtM communications. The aim of this letter is to extending previous results [6], [7], by proposing a couple of Modified NC (MNC) schemes which significantly outperform the Random Linear NC (RLNC) alternative considered in [7] both in terms of the transmission energy cost and delivery delay of the whole data flow. It is worth noting that the proposed communication methods require that all the UEs of the MG have to recover the transmitted flow with a decoding probability which is equal to or grater than a given threshold value.

The proposed MNC schemes achieve the aforementioned goals in two ways: (i) by optimally selecting the transmission data rate, while the power associated to transmission of each packet is kept constant, or (ii) by optimizing the transmission power cost and keeping constant the transmission rate. In the rest of the letter we will refer to those strategies as Constant Power MNC (CP-MNC) and Constant Rate MNC (CR-MNC), respectively. In addition, a theoretical framework for characterizing and efficiently optimizing both the proposed approaches will be presented.

The letter is organized as follows: Sec. II characterizes the system model, while the MNC-based communication strategies are presented in Sec. III. The analytical results are provided in Sec. IV. Finally, in Sec. V the conclusions are drawn.

II. System Model

A PtM communication pattern typical of a 3G/4G network can be modelled according to the multicast network model where a Base Station (BS) delivers a multicast service to a set of M UEs forming the MG. The NC communication strategy considered in this paper can be summarised as follows.

Let $\mathbf{E} = \{e_1, e_2, \ldots, e_K\}$ be an information message composed by K information packets. The BS linearly combines (in a *rateless* mode) all the information packets and transmits a stream of $N \geq K$ coded packets to the UEs of the MG. In the following, the widely known RLNC strategy [5] will be considered. As a consequence, the i-th (for $i = 1, \ldots, N$) coded packet can be defined as $c_i = \sum_{j=1}^{K} g_j \, e_j$, where the coefficients g_j are randomly chosen within the finite field $\mathcal{F}_q$ (of size q).

Whenever an UE successfully receives (at least) K linearly independent coded packets over N, it can retrieve the original information message $\mathbf{E}$. Conversely, if the number of the received and linearly independent coded packets is less than K the decoding operation fails, the UE can not retrieve the original information message, and it is definitively lost because retransmissions of the same information message are not allowed[1].

It is worth noting that in the case of the CP-MNC scheme each coded packet is transmitted at a rate that is m times smaller than the maximum allowed one (with $m \geq 1$) such that the symbol time duration is m times greater than the nominal value T. On the other hand, the CR-MNC scheme requires that transmission energy of each symbol is m times greater than a target value E (i.e., the power associated to the transmission of each coded packet is m times greater than a nominal value). Differently to CP-MNC, in this case the transmission rate does not change.

In spite of the increasing of the transmission time or transmission energy associated to each coded packet, this letter

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[1] We assumed that the BS can not know if a specific UE has actually received an information message because the reception of information messages can not be acknowledged.

will show that is possible to outperform the RLNC scheme, by means of a suitable optimization of the pair (m, N), both in terms of the overall delivery delay and energy consumption needed to complete the transmission of the set of N coded packets, for a given decoding probability of the information message (on a MG basis).

III. Optimal Transmission Rate and Energy Selection for NC-based Multicast Communications

Let us consider again the multicast network model presented in Sec. II, we have assumed slow-faded Rayleigh propagation conditions for all the communication channels linking each UE (of the MG) to the BS. In addition to this, we have also assumed that losses occur independently among UEs of the same MG, and the use of a Binary Phase-Shift Keying (BPSK) modulation scheme[2] in transmission. Hence, in the case of the CP-MNC and CR-MNC approaches, the corresponding signals associated to the transmission of a generic coded packet (for $1 \leq i \leq L$) can be expressed as follows

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} d_i \cos\left(2\pi f_o t\right) & \text{for CP-MNC with } 0 \leq t \leq mT \\ \sqrt{\frac{2mE}{T}} d_i \cos\left(2\pi f_o t\right) & \text{for CR-MNC with } 0 \leq t \leq T \end{cases} \quad (1)$$

where E is the energy of a transmitted signal with a time duration equal to T, f_o is the carrier frequency, L is the number of bits forming each coded packet, and d_i is equal to $+1$ or -1 if the i-th bit of the coded packet is 1 or 0, respectively.

From (1), in the case of the CP-MNC method it is important to stress that the amplitude of the transmitted signal is kept constant independently on the value of m. Likewise, for what concerns the CR-MNC, the transmission rate is kept constant regardless of the value of m. In addition, let us assume that the channel fading is sufficiently slow to make possible an exact estimation of the phase shift of the received signal at each receiving end, and hence, an ideal coherent detection can be performed. According to this, and whether CP-MNC or CR-MNC schemes are adopted, it is straightforward to prove that the bit error probability, as function of m, can be expressed as [8], [9]:

$$p_u(m) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{m\gamma_u}\right) \quad \text{for } u = 1, \ldots, M \quad (2)$$

where γ_u is the instantaneous Signal-to-Noise Ratio (SNR) related to the signal received by the u-th UE. Hence, according to the assumption of a Rayleigh fading, it follows that γ_u has a chi-square probability distribution with two degrees of freedom and a mean value equal to $\overline{\gamma}_u$. In addition, we have that the packet error probability (for $u = 1, \ldots, M$) for a L bits long coded packet can be expressed as[3]

$$P_u(m) = 1 - \int_0^\infty \frac{1}{\overline{\gamma}_u}\Big[1 - p_u(m)\Big]^L e^{-\frac{1}{\overline{\gamma}_u}\gamma_u} \; d\gamma_u \; . \quad (3)$$

[2]However, the theoretical results presented in this letter are quite general, as they can be easily extended to different modulation schemes.

[3]Note that the analysis outlined here is quite general and it includes also the case of different $\overline{\gamma}_u$ values among the UEs of the MG.

Let us define from [10] the probability that the u-th UE recovers the information message after that the BS has transmitted N coded packets (where $N \geq K$):

$$F_u(m, N) = \sum_{j=K}^{N} \binom{N}{j} P_u^{N-j}(m)\Big[1 - P_u(m)\Big]^j g(j) \quad (4)$$

where

$$g(j) = \prod_{h=0}^{K-1}\left[1 - \frac{1}{q^{j-h}}\right] \quad (5)$$

which is the probability that at least K coded packets are linearly independent over j (for $j \geq K$) [10].

Hence, for a MG formed by M UEs, the probability that *all* the UEs of the MG are able to recover the overall information message upon the transmission of N coded packets results to be

$$\Phi(m, N) = \prod_{u=1}^{M} F_u(m, N) \; . \quad (6)$$

Moreover, as for the parametric function $\hat{\Phi}_{\hat{N}}(m) = \Phi(m, N)\Big|_{N=\hat{N}}$ (for $\hat{N} \geq K$), it is a monotonically non-decreasing function. It is straightforward to note that the aforementioned statement holds because of the reasons below: (i) from (6) we have that $\hat{\Phi}_{\hat{m}}(N)$ is the product of the parametric functions $F_u(m, N)\Big|_{N=\hat{N}}$ (for $u = 1, \ldots, M$) which clearly are nonnegative nondecreasing functions[4], (ii) since the product of nondecreasing functions is nondecreasing too, the parametric function $\hat{\Phi}_{\hat{N}}(m)$ is nonnegative nondecreasing as well.

Let us consider the CP-MNC strategy, we have that $m\,L\,E$ and $m\,L\,T$ is the energy associated to the transmission of a single coded packet and its transmission time, respectively. On the other hand, in the case of CR-MNC the transmission time duration of a coded packet does not change, hence, it is equal to $L\,T$. In addition, in this case the energy cost associated to the transmission of a coded packet results to be equal to $m\,L\,E$. Hence, for both the considered approaches, we have that the overall transmission energy cost, i.e., the overall energy associated to the transmission of N coded packets, is

$$\epsilon(m, N) = m\,L\,E\,N \quad (7)$$

while the delivery delay, i.e., the overall time needed to transmit N coded packets, is

$$\delta(m, N) = \begin{cases} m\,L\,T\,N & \text{for CP-MNC} \\ L\,T\,N & \text{for CR-MNC.} \end{cases} \quad (8)$$

From above, it is straightforward to note that the case of $m = 1$ is related to the RLNC scheme [5] (where both the transmission rate and power are kept constant).

A. CP-MNC Optimization

Let us focus on the CP-MNC strategy. In this Section, we will propose an analytical procedure which aims to optimize the CP-MNC approach. The optimization procedure aims to derive the optimum values of the transmission rate (i.e., the

[4]Due to the fact that the probability that the u-th UE recovers the overall information message can not decrease as the packet error probability decreases. In particular, it is straightforward to note that $P_u(m)$ is a non-decreasing function.

optimum value of m) and N in order to minimize the overall transmission energy cost and delivery delay, if compared with the RLNC scheme (where m is always equal to 1). In particular, from (7) and (8) it is worth noting that the goal is achieved by minimizing the object function given by $m\,N$. Hence, our optimization problem can be formulated as follows[5]:

$$\text{(O1)} \qquad \min_{m,N} \; m\,N \tag{9}$$

$$\text{subject to} \quad \Phi(m,N) \geq \hat{\Phi} \tag{10}$$

$$1 \leq m \leq \hat{m}_{max}, \quad m \in \mathbb{R}^{+} \tag{11}$$

$$K \leq N \leq \hat{N}_{max}, \quad N \in \mathbb{N} \tag{12}$$

where $\hat{\Phi}$ is the target delivery probability of an information message, i.e., the probability that all the UEs of the MG recover the whole information message. In addition, due to the fact that the transmission time duration and energy associated to an information message are function of m and N, it follows that both m and N have practical upper bounds which dependent on the specific QoS constraints. These practical limits are modelled by constraints (11) and (12) where we assumed that the parameter $\hat{m}_{max}$ (for $\hat{m}_{max} \geq 1$) and $\hat{N}_{max}$ (for $\hat{N}_{max} > K$) is the upper bound for m and N, respectively.

Due to the fact that O1 is a mixed integer non-linear optimization problem, it can not be solved with reasonable computing efforts. For this reason, the rest of this section shows how to transform O1 into an equivalent problem that can be efficiently solved. In particular, in the rest of the Section we will show that O1 can be solved by a two-steps procedure which can be summarised as follows: (i) for each $N \in [K, \hat{N}_{max}]$ the optimum value of m is found, then (ii) the pair (m,N) which minimises the objective (9) is selected.

Let us consider the problem O1, it can be rewritten as

$$\text{(O2)} \qquad \min_{N} N \; \min_{m} m \tag{13}$$

$$\text{subject to} \quad m \in \mathcal{S}_N \tag{14}$$

where the set $\mathcal{S}_N$ is defined as $\mathcal{S}_N \doteq \left\{ m \in \mathbb{R}^{+} \;\middle|\; 1 \leq m \leq \hat{m}_{max} \;\wedge\; \hat{\Phi}_N(m) \geq \hat{\Phi} \right\}$. The problem O2 is a *nested* optimization model because it consists of two sub-problems. In particular, for $N = N^{*}$ the innermost problem is

$$\text{(I1)} \qquad \min_{m} m \tag{15}$$

$$\text{subject to} \quad m \in \mathcal{S}_{N^{*}}\,. \tag{16}$$

In addition, let $\mathcal{S}$ be the feasible set of O1, of course, the relation $\mathcal{S}_{N^{*}} \subseteq \mathcal{S}$ holds. In particular, due to the fact that $\hat{\Phi}_{N^{*}}(m)$ is a monotonically non-decreasing function, the solution of I1 is the smallest value of m (for $m \geq 1$) such that the relation $\Phi(m,N^{*}) \geq \hat{\Phi}$ holds. As a consequence, the problem O2 can be solved by the two-steps procedure below:

i For each $N = K, \ldots, \hat{N}_{max}$ find $m_K, m_{K+1}, \ldots, m_{\hat{N}_{max}}$ which are the solutions to the problem I1 for $N^{*} = K, K+1, \ldots, \hat{N}_{max}$.

ii The solution of O2 (i.e., the optimal (m,N) pair) is the pair (m_i, i), for $i = K, K+1, \ldots, \hat{N}_{max}$, minimizing the objective function (9).

[5] In this paper we refer with $\mathbb{R}^{+}$ and $\mathbb{N}$ to the set of real positive and natural numbers, respectively.

B. CR-MNC Optimization

Let us consider the CR-MNC approach. In this case the value of m has a direct impact on the power associated to the transmission of each coded packet. Hence, also in this case the value of m has a practical upper bound (which is $\hat{m}_{max}$) depending on the system in use. In addition, we remark that we assumed $m \geq 1$.

Due to the fact that also the CR-MNC method aims to deliver each information message in such a way that it can be received by the considered MG at least with a certain delivery probability $\hat{\Phi}$, the optimization of the pair (m,N) takes places over the same kind of feasible set associated to O1. However, in this case we decided to minimize the overall transmission energy cost, i.e., we decided to minimize the overall power cost associated to the transmission of each information message. Hence, from (7), it is straightforward to note that, also in this case the model O1 can be efficiently use to achieve the aforementioned goals. Hence, the optimum pair (m,N) can be found by the two-steps procedure proposed in Sec. III-A. Finally, in Sec. IV we will show that not only the overall transmission energy cost is minimized but also the delivery delay is significantly reduced if compared to the RLNC alternative.

Finally, from (7) and (8) it is straightforward to note that the overall transmission energy cost and delivery delay can not be jointly minimized. In particular, if only the delivery delay is minimized, the value of m will trivially be equal to $\hat{m}_{max}$, i.e., the power associated to the transmission of each coded packet is maximized.

IV. Numerical Results

The performance of both CP-MNC and CR-MNC strategies optimized as proposed in the previous section are hereafter evaluated. We investigate the system performance in terms of the normalized overall transmission energy cost and delivery delay, defined as $\epsilon(m,N)/(K\,L\,E)$ and $\delta(m,N)/(K\,L\,T)$, respectively. From (7) and (8) we have that, in the case of CP-MNC, the normalized overall transmission energy cost and delivery delay are equal.

In addition, we compare the MNC-based strategies to an RLNC approach where the value of N has been optimized by the flowing model (which is based on O1):

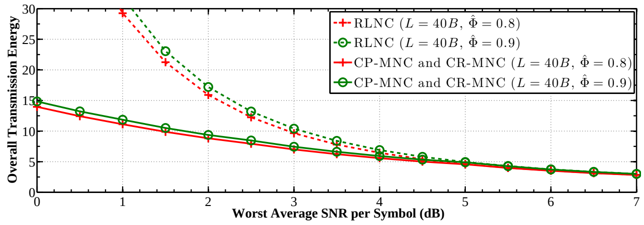
$$\text{(O3)} \qquad \min_{N} N \tag{17}$$

$$\text{subject to} \quad \Phi(m,N)\Big|_{m=1} \geq \hat{\Phi} \tag{18}$$

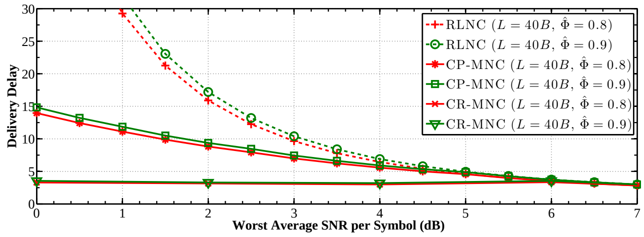
$$K \leq N \leq \hat{N}_{max}, \quad N \in \mathbb{N}\,. \tag{19}$$

It is worth noting that in the case of RLNC, the transmission time duration, energy and power associated to each coded packet never change (i.e., the value of m is always equal to 1).

The scenarios herein considered refer to a MG composed by a variable number of UEs (namely, $M \in [2, 128]$). We assumed a finite field size equal to 2^8 for all the NC-based



(a) Normalized overall transmission energy cost



(b) Normalized overall delivery delay

Fig. 1. Normalized overall transmission energy cost and delivery delay vs. average SNR of the UE experiencing the worst propagation conditions.

coding/decoding operations and a message length of $K = 20$ information packets. Two different packet lengths ($L = 20$ bytes and $L = 40$ bytes) have been considered. Finally, regardless to the considered MNC-based strategy, parameters $\hat{m}_{max}$ and $\hat{N}_{max}$ have been set equal to 10 and $50\,K$, respectively.

Fig. 1a and Fig. 1b show, for $M = 20$, the values of $\epsilon(m, N)/(K\,L\,E)$ and $\delta(m, N)/(K\,L\,T)$ as function of $\overline{\gamma}_w$ which is the average SNR value of the UE experiencing the worst propagation conditions. In this scenario each UE is associated to an average SNR $\overline{\gamma}_u \in [0, 10]$ dB (for $u = 1, \ldots, M$). The figures show the performance metrics for two different values of the target delivery probability, namely $\hat{\Phi} = 0.8$ and $\hat{\Phi} = 0.9$.

As for the normalized overall transmission energy cost, Fig. 1a clearly shows a maximum performance gain of at least three-fold for both the proposed CP-MNC and CR-MNC methods if compared to the optimized RLNC scheme. On the other hand, Fig. 1b shows that the proposed MNC-based strategies outperform the optimized RLNC scheme in terms of the normalized delivery delay. In particular, it is worth noting that in this case the maximum normalized delivery delay which characterises the CP-MNC and CR-MNC methods is at least three-times and eight-times smaller than that associated to the optimized RLNC, respectively.

Even though the CR-MNC method outperforms the CP-MNC alternative in terms of the delivery delay, this result is achieved at the cost of an increase of the transmission power. However, as the overall power consumption at the BS side increases, the interference to other UEs or networks (which operates on the same frequency band) augments. Of course, if the transmission power cost is minimized, also the impact of the interference on the overall system performance can be significantly reduced.

Finally, Fig. 2 compares the performance of the considered strategies as function of M. The figure refers to a network scenario where all the UEs are charactered by the same average SNR per symbol which is 1 dB. Fig. 2 shows that

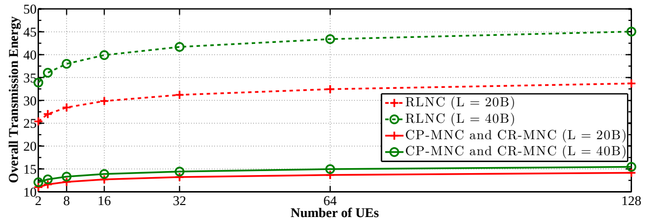


Fig. 2. Normalized overall transmission energy cost vs. number of UEs.

the normalized overall transmission energy cost associated to the MNC-based strategies is up to three-times smaller than that associated to the optimized RLNC alternative.

V. Conclusion

This paper has proposed the CP-MNC and CP-MNC multicast communication schemes which allow the efficient transmission of information flows by resorting to the optimization of the transmission rate, transmission energy and number of delivered coded packets. The proposed MNC-based strategies minimise the overall transmission energy cost and significantly reduce the delivery delay in comparison to the RLNC alternative. In particular, a performance gain of at most three-fold in overall transmission energy cost and delivery delay is achieved by the CP-MNC method if compared to the optimized RLNC alternative. For what concerns the CR-MNC method, its maximum overall transmission energy cost and delivery delay are respectively at least three-times and eight-times smaller than those of the optimized RLNC. Finally, we stress that there is no simple answer to the problem of choosing among the proposed methods. The choice of the CP-MNC method as well as of the CR-MNC one depends not only on benefits/drawbacks associated to each alternative but also on the specific delivered services and performance criteria of interest.

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