Optimized Short Message Transmission for Reliable Communications over Satellite Broadcast Channels

Francesco Chiti, Romano Fantacci, Tommaso Pecorella, Andrea Tassi

Department of Information Engineering University of Florence, Italy

Email: {francesco.chiti,romano.fantacci,tommaso.pecorella,andrea.tassi}@unifi.it

Abstract—This paper deals with the issue of reliably transmission of short messages through satellite. The problem is of paramount importance for emergency and alerting scenarios. In particular, we propose a novel efficient Network Coding (NC) communication scheme aiming to improve the delivery probability of the transmitted information messages. A suitable analytical approach has been developed in order to highlight the performance of the proposed NC scheme and to allow its optimization. The accuracy of the proposed approach has been validated by resorting to computer simulations. Performance comparisons with the classical NC scheme are also presented here to highlight the advantages of the proposed NC scheme in the case of AWGN and Rician communication channels.

I. INTRODUCTION

The Network Coding (NC) principle has been introduced by Ahlswede *et al.* [1] as a new communication strategy able to remarkably increase the communication throughput of wired networks. Afterwards, the NC has been applied also to wireless networks in [2], [3] with the aim of increasing the communication throughput in multi-hop topologies or to increase the data reliability at the receiving end [4].

In particular, in the field of satellite networks, reliability of broadcast communication [5] has been addressed by means of Forward Error Correction (FEC) codes. Modern systems can have a feedback channel, e.g., DVB-RCS or DVB-RCS2, allowing the adoption of an ARQ-based error control protocol. However, the existing approaches are not suitable for short-duration communications, for e.g., traffic information, meteorological conditions and alerting systems, as in this case the message-based services are usually characterized by short, sporadic messages that have to be broadcasted reliably to a large number of clients [6] and the ARQ-based systems suffer from scalability issues. This can be overcome by using the NC principle as error control strategy [7].

Differently from the approaches previously proposed in the literature, this paper proposes a Modified NC (MNC) scheme where the transmission of each packet is iteratively repeated m times, with m assumed as a system parameter. It is straightforward to note that this is equivalent to increase the duration of each symbol of a factor m with respect to the classical case [8]. In this paper we assumed that the m copies of the same packet are soft-combined symbol-by-symbol by each user during the receiving phase, then the network client decides if the packet has been correctly received (or not).



Fig. 1. The considered network model.

As will be highlighted in the following, the performance of the MNC scheme depends on m, so that this paper proposes: (i) an optimization approach based on the definition of a proper target function, and (ii) an ideal and heuristic optimization approach. In particular, the latter scheme has been proved to be convex. In order to highlight the advantages of the proposed MNC scheme, a performance comparison between the MNC and classical NC has been provided.

The paper is organized as follows. Section II provides the necessarily backgrounds on the NC principle. The proposed NC-based optimized communication scheme is described in Section III. Numerical results are provided in Section IV. Finally Section V gives the conclusions of the paper.

II. SYSTEM MODEL

In this paper we will consider a Geostationary satellite (GEO) system (sketched by Fig. 1) where a Control Centre (CC) can send short data messages to M users (namely, W_1, W_2, \ldots, W_M) spread across a region by mean of the satellite system. It is outside of the scope of the present work to identify the particular satellite standard, as the proposed approach can work with any satellite system able to send short messages.

It is worth noting that the satellite transmits each information message (i.e., the short messages) by using the NC principle (Sec. II-A). Whenever a receiving node has successfully recovered an information message, it sends back an acknowledgement¹ (ACK) to the satellite. When all the

¹We have assumed here, without any loss of generality, that ACK messages are sent across a fully reliable channel.

nodes have successfully recovered the same message, the satellite can start the transmission of a new one. For the sake of the analysis, we assumed that NC operations are implemented on-board the satellite. However, the provided theoretical framework remains valid even when the NC process is implemented in the CC.

In carrying out our analysis we have referred to the Quadrature Phase-Shift Keying (QPSK) modulation scheme² for the data transmission in the considered scenarios.

A. Network Coding communication scheme

Ghaderi *et al.* [4] showed that the NC principle can be used to implement an error control strategy for Point-to-Multipoint (PtM) communication flows. In particular, the transmitting node delivers to a receiving one a message (the so-called generation) $\mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N]$ consisting of N packets (the generation length). The transmitting node iteratively broadcasts coded packets obtained (in a rateless fashion) as linear combinations of elements belonging to the same generation **E**. From the main theorem of NC [9] it follows that a receiving node can recover the original message **E** only if N (at least) linearly independent coded packets have been successfully received.

The coding process (i.e., the computation of a coding packet $\hat{\mathbf{e}}_j$) can be defined as $\hat{\mathbf{e}}_j = \mathbf{c}_j \cdot \mathbf{E}^T$, where the *N*-dimensional row vector \mathbf{c}_j represents the *j*-th coding vector. In this paper we will refer to the widely adopted Random Linear Network Coding (RLNC) [9] scheme, where each component of the coding vector is randomly chosen within a finite field \mathcal{F}_q of size *q*. Hence, the probability that two coding vectors are linearly dependent is nonzero. For this reason, the transmitting node needs to broadcast at least *G* (with $G \ge N$) coded packets, in order to ensure the correct reception (at the receiving end) of *N* linearly independent coded packets³. In the rest of the paper, without loss of generality, we will assume that coding vectors are known at receiving ends.

It is worth noting that with the MNC strategy each coded packet is transmitted m times by the satellite. We remark that each network client has to soft-combine (symbol-by-symbol) all the copies of the same coded packet in oder to recover it. In the next Section we will propose an optimization model for the system parameter m aiming to minimize average transmission delay of each information message.

III. MODIFIED NETWORK CODING PRINCIPLE IN SATELLITE BROADCAST COMMUNICATION NETWORKS

This Section deals with the performance evaluation of the proposed MNC scheme by considering the broadcast network topology (sketched in Fig. 1), under AWGN and Rician propagation conditions. In particular, a suitable optimization approach will be proposed here in order to derive the optimal value of m allowing the best performance in terms of the mean overall communication delay.

A. AWGN regime

Let us assume that: (i) coded packets are transmitted over M independent AWGN channels, and (ii) at each receiving end losses of coded packets occur as statistically independent events. Let $\gamma = \frac{E_b}{N_0}$ be the Signal-to-Noise Ratio (SNR) per symbol characterizing the reception of each node. The parameter E_b is the energy associated to each transmitted symbol and N_0 is the one side AWGN spectral density. We remark that each node is characterized by the same value of γ . In the case of QPSK modulation we have that the bit error probability can be expressed as follows [8], [10]:

$$P_e(m,\gamma) = Q\left(\sqrt{m\,\gamma}\right) \tag{1}$$

where $Q(\cdot)$ is the well known Q-function. From Eq. (1), the delivery probability of a coded packet (*L* bits long) can be defined as follows:

$$P_C(m) = \left[1 - P_e(m, \gamma)\right]^L \,. \tag{2}$$

Let m and N be the chosen BDI factor and the generation length, respectively. Let S_i (for i = 1, ..., M) be a random variable representing the number of transmission attempts performed by the satellite to ensure the correct reception of an information message by W_i (i.e., to ensure the correct reception of N linearly independent coded packets). Moreover, let

$$H = \max_{i=1,\dots,M} \left\{ S_i \right\} \tag{3}$$

be the random variable representing the number of transmission attempts performed by the satellite to ensure the correct reception of a generation by all the receiving nodes. The function expressing the probability that $S_i \leq j$ (for $j \geq N$) can be expressed as [11], [12]:

$$f_i(j) = \sum_{a=N}^{j} {j \choose a} P_C^a(m) \Big[1 - P_C(m) \Big]^{j-a} p_{NC}(a, N) \quad (4)$$

where

$$p_{NC}(a,N) = \prod_{b=0}^{N-1} \left[1 - \frac{1}{q^{a-b}} \right].$$
 (5)

The $p_{NC}(a, N)$ term is the probability that at least N over $a \ge N$ coded packets (belonging to the same generation) are linearly independent [11]. For j < N, $f_i(j)$ is null.

From Eqs. (3) and (4), the average value of the random variable H can be expressed as [13]:

$$\zeta_{NC}(m) = \sum_{n=0}^{\infty} n \left\{ \operatorname{Prob}\left\{H \le n\right\} - \operatorname{Prob}\left\{H \le n-1\right\} \right\} = \sum_{n=N}^{\infty} n \left\{ \prod_{r=1}^{M} f_r(n) - \prod_{r=1}^{M} f_r(n-1) \right\}.$$
(6)

From Eq. (6) we define the mean broadcast delay as the mean time needed by all the network nodes to successfully receive an information message (N packets long). It can be expressed as follows:

$$\Lambda_{NC}(m) = m \,\zeta_{NC}(m) \,T_s \tag{7}$$

²Results provided in this paper are quite general and can be easily extended to different modulation schemes, like 8PSK, 16APSK, or 32APSK, etc.

³We omit in this Section the channel effects on the received coded packets.

where T_s is the mean propagation delay of a coded packet when the BDI factor is equal to one (i.e., m = 1). In order to make our analysis general, we refer in what follows to the mean broadcast delay ($\Lambda_{NC}(m)$), normalized with respect to the parameter N and T_s :

$$\Lambda_{NC}(m) = \frac{\tilde{\Lambda}_{NC}(m)}{N T_s} = \frac{m \zeta_{NC}(m)}{N} .$$
(8)

Further, it is important to note that the normalized mean broadcast delay of the classical RLNC scheme is equal to $\Lambda_{NC}(1)$, i.e., it is equal to $\Lambda_{NC}(m)$ for m = 1.

As a consequence, we can derive the optimal BDI factor by solving the following optimization problem⁴:

(oMNC) minimize
$$\Lambda_{NC}(m)$$
 (9)

subject to
$$m \in \mathbb{N}$$
 (10)

The oMNC is an integer nonlinear optimization problem whose solution is hard to derive in a closed form, hence we need to resort to derivative-free methods⁵.

As an alternative to the previous approach, we propose to consider a novel heuristic model. It is characterized by a reduced computational complexity, and at the same time, the obtained solution is close enough to the optimal one (it will be shown in Sec. IV). It is useful to define the mean link delay $(\hat{\lambda}(m))$ as: the mean time required by a given receiving node to successfully collect an arbitrary number $G \ge N$ of coded packets. It can be expressed by:

$$\hat{\lambda}(m) = \frac{m G T_s}{P_C(m)} .$$
(11)

Let \mathcal{P} be the set of real numbers equal to or greater than one; the mean link delay function, normalized to $G T_s$, $\Lambda_{L2L}(m) : \mathbb{N} \to \mathcal{P}$ results to be:

$$\Lambda_{L2L}(m) = \frac{\lambda(m)}{GT_s} = \frac{m}{P_C(m)} .$$
 (12)

Hence, from Eq. (12), the BDI factor can be optimized by the following heuristic approach:

(hMNC) minimize
$$\Lambda_{L2L}(m)$$
, (13)

subject to
$$m \in \mathbb{N}$$
 (14)

In order to solve the hMNC optimization problem, let us consider the following proposition:

Proposition 1: Let $\hat{\Lambda}_{L2L}(\hat{m}) : \mathcal{P} \to \mathcal{P}$ be the continuous expansion of the function $\Lambda_{L2L}(m) : \mathbb{N} \to \mathcal{P}$. The function $\hat{\Lambda}_{L2L}(\hat{m})$ is continuously differentiable and convex on its domain.

Proof: See the Appendix.

In order to solve the hMNC problem, let us consider the following one:

(rMNC) minimize
$$\Lambda_{L2L}(\hat{m})$$
, (15)

subject to
$$\hat{m} \in \mathcal{P}$$
 (16)

 $^{4}\mathrm{In}$ the paper we will refer to the set of positive and non-null integer numbers as $\mathbb N.$

⁵In particular, in this paper we have resorted to the NOMAD solver [14].

Since $\hat{\Lambda}_{L2L}(\hat{m})$ is convex in its domain see (Proposition 1), the solution (\hat{m}_o) of the rMNC problem is the real root (if it exists) of the following equation:

 $\frac{d}{d\hat{m}}\left(\hat{\Lambda}_{L2L}(\hat{m})\right) = 0 \Leftrightarrow w(\hat{m}) - t(\hat{m}) = 0$

$$w(\hat{m}) \doteq 1 - Q(\sqrt{\hat{m}\gamma}) , \qquad (18)$$

(17)

$$t(\hat{m}) \doteq L \sqrt{\frac{\gamma \, \hat{m}}{2\pi}} e^{-\frac{\hat{m} \, \gamma}{2}} \,. \tag{19}$$

In particular, the solution (m_o) of the hMNC problem is represented by $\lceil \hat{m}_o \rceil$ or $\lfloor \hat{m}_o \rfloor$. Hence, the hMNC problem can be easily solved by choosing the value minimizing the Eq. (13). If Eq. (17) has no real root, the solution of the hMNC problem is $m_o = 1$. Even though a closed-form solution to the Eq. (17) is not achievable, its convexity ensures that the rMNC problem can be efficiently solved by resorting to suitable numerical approaches [15]. In particular, we have resorted here to the CVX solver [16].

B. Rician regime

where

In order to point out the strengths of the MNC, let us consider the case of a slow Rician fading regime. In particular we assumed that: (i) propagation conditions are kept constant during the transmission of m copies of the same coded packet (regardless to the value of m), (ii) the M communication channels are independent, and (iii) by using suitable radio resource allocation scheme, they can be considered statistically independent. For these reasons we can assume that losses of coded packets at each receiving end occur as statistically independent events. Due to the fact that the channel fading is slow, we can assume an ideal coherent detection at each receiving node side.

Let $\gamma_i = \alpha_i^2 \frac{E_{b,i}}{N_0}$ be the Signal-to-Noise Ratio (SNR) at the W_i side where: α_i^2 is a noncentral χ^2 -distributed random variable (with two degrees of freedom) representing the squared magnitude of a Rice channel coefficient (α_i). The parameter $E_{b,i}$ is the energy associated to each symbol received by W_i . The probability density function of γ_i [17] is given by

$$r(\gamma_i) = \left(\frac{1+K}{\overline{\gamma}}\right) e^{-\frac{(1+K)\gamma_i + K\overline{\gamma}}{\overline{\gamma}}} I_0\left(2\sqrt{\frac{K(1+K)\gamma_i}{\overline{\gamma}}}\right)$$
(20)

where $\overline{\gamma}$ is the average SNR characterizing each receiving node, K is the well known Rician parameter [17], and $I_0(\cdot)$ is the 0-th order modified Bessel function of the first kind. For these reasons the $P_C(m)$ in this case can be expressed as:

$$P_C(m) = \int_0^\infty \left[1 - P_e(m, \gamma) \right]^L r(\gamma) \ d\gamma \ . \tag{21}$$

Also in this case the BDI factor can be optimized by the oMNC model. This problem cannot be solved with affordable computing efforts⁶. However, the Proposition 1 holds also in

⁶As in the AWGN scenario, the oMNC problem has been solved by the NOMAD solver [14].



Fig. 2. Normalized mean broadcast delay of RLNC and MNC (optimized by the oMNC and hMNC models) vs. the SNR value (AWGN regime).



Fig. 3. Normalized mean broadcast delay of RLNC and MNC (optimized by the oMNC and hMNC models) vs the number of receiving nodes (AWGN regime).

the presence of Rician propagation conditions⁷. Hence, the rMNC problem is still a convex optimization problem. For this reason, it can be efficiently solved by suitable numerical approaches⁸. In particular, the solution (\hat{m}_o) of the rMNC is the real root (if it exists) of the following equation:

$$\frac{d}{d\,\hat{m}}\Big(\hat{\Lambda}_{L2L}(\hat{m})\Big) = 0 \ . \tag{22}$$

It can be proved that the Eq. (22) can be restated as reported by Eq. (17) where in this case:

$$w(\hat{m}) \doteq \int_0^\infty \left[1 - Q(\sqrt{\hat{m}\gamma}) \right]^L r(\gamma) \ d\gamma \tag{23}$$

and

$$t(\hat{m}) \doteq \frac{L\sqrt{\hat{m}}}{2\sqrt{2\pi}} \int_0^\infty \sqrt{\gamma} \left[1 - Q(\sqrt{\hat{m}\gamma}) \right]^{L-1} e^{-\frac{\hat{m}\gamma}{2}} r(\gamma) \ d\gamma.$$
(24)

Hence, the solution (m_o) of the hMNC problem is that value minimizing Eq. (13) chosen between $\lceil \hat{m}_o \rceil$ and $\lfloor \hat{m}_o \rfloor$. If Eq. (22) does not have any solution, the m_o is equal to 1.



Fig. 4. Normalized mean broadcast delay of RLNC and MNC (optimized by the oMNC and hMNC models) vs. the average SNR value (Rician regime).



Fig. 5. Normalized mean broadcast delay of RLNC and MNC (optimized by the oMNC and hMNC models) vs. the number of receiving nodes (Rician regime).

IV. NUMERICAL RESULTS

In this Section, by resorting to computer simulations, a performance comparison between the classical RLNC and the optimized MNC schemes will be proposed. We have considered two different scenarios:

- I The satellite transmits packets 42 or 21 bytes long to 20 receivers (i.e., M = 20). In the case of AWGN (Rician) propagation conditions, each receiving node is characterized by the same SNR (average SNR) value $\gamma \in [0, 10]$ dB ($\overline{\gamma} \in [0, 10]$ dB);
- II The satellite transmits to a variable number of nodes packets 42 bytes long. Also in this case, we have that each node is characterized by the same SNR (mean SNR) value: $\gamma = 7.5$ dB in the case of the AWGN regime and $\overline{\gamma} = 5$ dB for the Rician faded channel.

Moreover, we assumed for all the RLNC-based communications the finite field size $q = 2^2$ (i.e., all the items of coding vectors are 2 bits long) and a generation length of 20 information packets. Finally, in the case of the Rician regime, we set the parameter K to 5 dB. We shall stress out that the

⁷It will be proved in the Appendix.

⁸Also in this case the rMNC problem has been solved by resorting to the CVX solver [16].

packet length is arbitrary and the results can be extend to any packet length.

Fig. 2 shows the normalized mean broadcast delay as function of the SNR value. This figure refers to the network scenario I in AWGN propagation conditions. On the other hand, Fig. 3 shows the same performance metric as function of the number of receiving nodes in the network scenario II (characterized by the same propagation conditions). We can note that both the oMNC and hMNC models minimize the normalized mean broadcast delay of the MNC scheme if compared to the RLNC. For instance, Fig. 3 shows that in a network composed by M = 1024 users, the MNC (optimized by the hMNC model) is characterized by a (normalized) mean broadcast delay that is almost 12-fold smaller than that of the RLNC. In addition to that, the hMNC model is characterized by almost the same performance that we would have by using the oMNC model.

Fig. 4 and 5 show the same performance metrics reported by Fig. 2 and 3 in the case of a Rician faded channel, for the network scenarios I and II, respectively. Also in this case we can note that: (i) the optimized MNC scheme achieves a performance gain of almost 5-fold if compared to the classical RLNC (if we consider the Fig. 5 for M = 1024 users), and (ii) the performance of the hMNC model is very close to that of the oMNC model.

V. CONCLUSION

In this paper we proposed an improvement to the RLNC scheme by resorting to an optimization strategy that proactively increases the symbols duration with the aim of minimizing the overall communication delay. By resorting to computer simulations, a performance comparison between the convex optimization model (hMNC) and the optimal one (oMNC) has been carried out. The numerical results provided here compare the oMNC and hMNC models in terms of the normalized mean broadcast delay. Afterwards, it has been shown that the reduced complexity hMNC scheme is a good approximation of the oMNC scheme. In addition to that, the numerical results show that the optimized MNC scheme is characterized by a performance gain of almost 5-12 fold if compared to the classical RLNC scheme. We remark that the proposed approach can be efficiently used for delivery services without requiring any modifications of the satellite system. In addition, the presented theoretical framework can be used by service providers to predict the average transmission time of a message. The latter point is especially important for service provisioning and system sizing.

APPENDIX PROOF OF THE PROPOSITION 1

Proof: Due to their definitions, the functions $w(\hat{m})$ and $t(\hat{m})$ are continuously differentiable in \mathcal{P} (both for AWGN and Rician propagation conditions). Moreover, in the considered propagation regimes, the first-order derivative of $\hat{\Lambda}_{L2L}(\hat{m})$,

can be expressed as follows:

$$\frac{d}{d\hat{m}}\left(\hat{\Lambda}_{L2L}\right) = \frac{w(\hat{m}) - t(\hat{m})}{w^2(\hat{m})} .$$
(25)

In the case of AWGN propagation conditions, let us consider a packet length $L \ge 8$ bits and an SNR value $\gamma \ge 0$ dB. On the other hand, in the case of Rician propagation conditions, let us consider the following parameters: $L \ge 8$ bits, a mean SNR value $\overline{\gamma} \ge 0$ dB, and a Rician factor $K \ge 1$ dB. In both propagation regimes the relation $\frac{d^2}{d\hat{m}^2} (\hat{\Lambda}_{L2L}) \ge 0$ holds. Hence, $\frac{d}{d\hat{m}} (\hat{\Lambda}_{L2L})$ increases. For these reasons $\hat{\Lambda}_{L2L}(\hat{m})$ is convex in \mathcal{P} [15].

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