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Communication Systems & Networks Group

Sparse Random Network Coding for Reliable Multicast Services

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Starting Point and Goals

- Point-to-Multipoint communications play a pivotal role in 4G/5G networks. We champion RLNC.
- A fundamental problem: the computational complexity of the RLNC decoder. The higher the number of decoding operations is, the more the user's computational overhead grows and, consequently, the faster the batteries of mobile devices drain.
- Layered service consists of a **basic layer** and **multiple enhancement layers**.

Goals

- Efficient way to characterize the performance of users targeted by **ultra-reliable layered multicast services**
- **Convex RA framework** for minimizing the complexity of the RLNC decoder by jointly optimizing the **transmission parameters** and the **sparsity of the code**.

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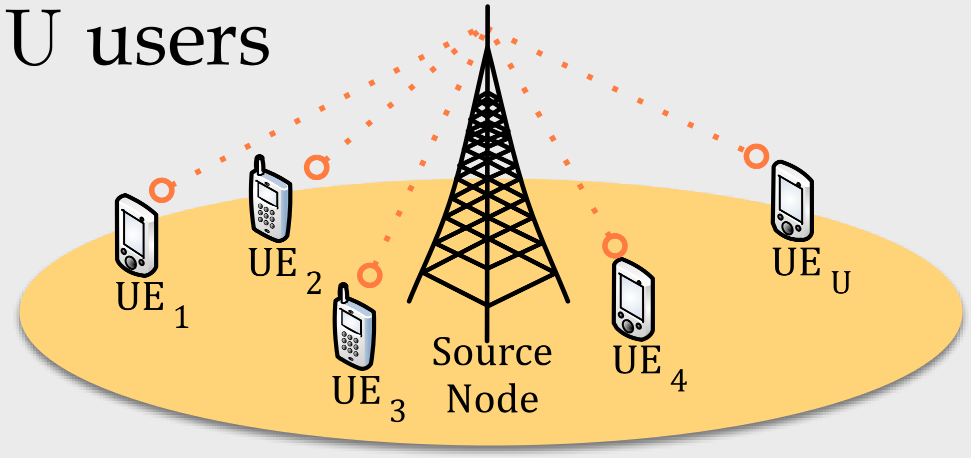
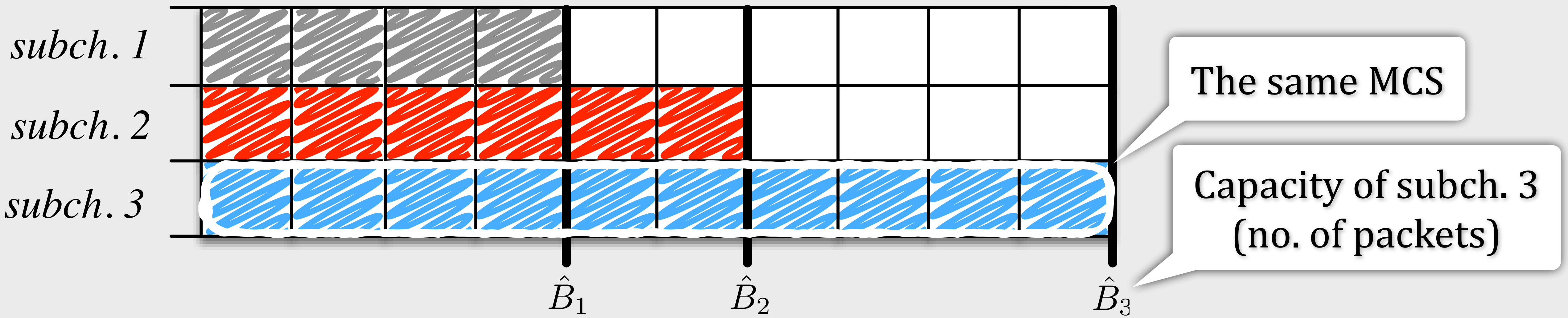
1. System Model and RNC Background

System Model

- One-hop wireless communication system composed of one source node and U users



- Each PtM layered service is delivered through C orthogonal broadcast erasure subchannels



- Each subchannel delivers streams of (en)coded packets (according to a RLNC scheme).

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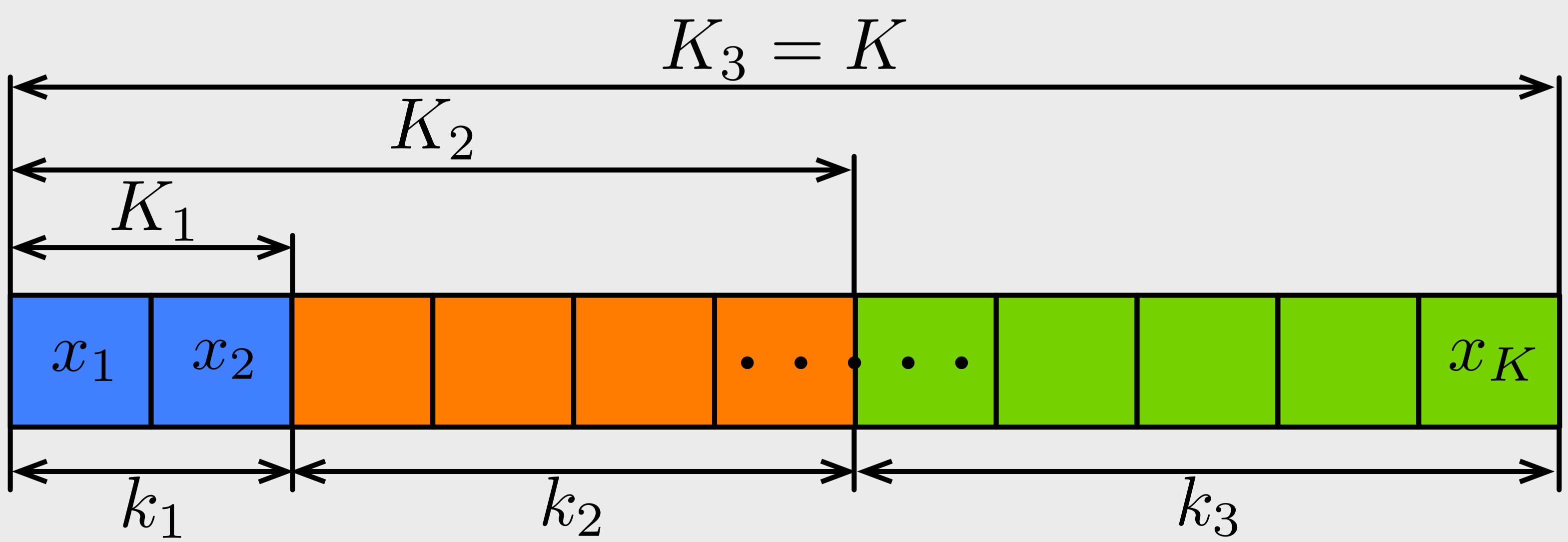
RNC in a Nutshell

- $\mathbf{x} = \{x_1, \ldots, x_K\}$ is a layered source message of K source packets, classified into L service layers



- Encoding performed over each service layer independently from the others.

- The source node will linearly combine the k_l data packets composing the l-th layer $\{x_i\}_{i=K_{\ell-1}+1}^{K_\ell}$ and will generate a stream of coded packets, where

$$y_j = \sum_{i=K_{\ell-1}+1}^{K_\ell} c_{j,i} \cdot x_i$$

Coefficients of the linear combination are selected over a finite field of size q

RNC in a Nutshell

- Let us refer to the user u and layer l. As user u successfully receives a coded packet, the corresponding coding vector is extracted and added, as a new row, into matrix $\mathbf{C}_u$.

- Assume u already received $n_\ell \geq k_\ell$ coded packets

n_ℓ rows x k_ℓ cols

$$\mathbf{C}_u \doteq \begin{bmatrix} \boxed{\mathbf{c}_1} \\ \boxed{\mathbf{c}_2} \\ \vdots \\ \boxed{\mathbf{c}_{n_\ell}} \end{bmatrix}$$

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RNC in a Nutshell

- When $\mathbf{C}_u$ has rank equal to k_ℓ, the user can keep only the linearly independent rows and invert the matrix.

$$\boxed{\mathbf{C}_u \cdot \mathbf{x}_\ell^T = \mathbf{y}^T} \Longleftrightarrow \boxed{\mathbf{C}_u^{-1} \cdot \mathbf{y}_\ell^T = \mathbf{x}_\ell^T}$$

Encoding

Decoding

- Given [1, 0] and [0, 1], is [2, 2] linearly independent?
  - No, because 2[1, 0] + 2[0, 1] - [2, 2] = [0, 0]
- Given two lin. indep. vectors (**a** and **b**) in GF(q), how many vectors form *span*({**a**,**b**})?
  - q^2. For a = [1, 0], b = [0, 1] and q = 2, *span*({**a**,**b**}) = {[0, 0], [0, 1], [1, 0], [1, 1]]}
- Let us encode over a set of 5 inf. elements, if I collected 3 lin. indep. coding vectors, what is the prob. of collecting a new lin. dep. coding vec.?
  - q^3 / q^5

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2. Performance Analysis via Markov Chains

The Coding Matrix

- Matrix $\mathbf{C}_u$ is a random matrix over GF(q), where elements are independently and uniformly selected by the following prob. law

$$\Pr\left(c_{j,i} = v\right) = \begin{cases} p_\ell & \text{if } v = 0 \\ \dfrac{1 - p_\ell}{q - 1} & \text{if } v \in \mathbf{GF}(q) \setminus \{0\} \end{cases}$$

- If $p_\ell = 1/q$, all the GF(q) elements are equiprobable and things are nice and easy… otherwise, things get tricky!

- Since 1997, only 2 conference papers and 2 (+1 on Arxiv) journal papers deal with sparse random matrices.

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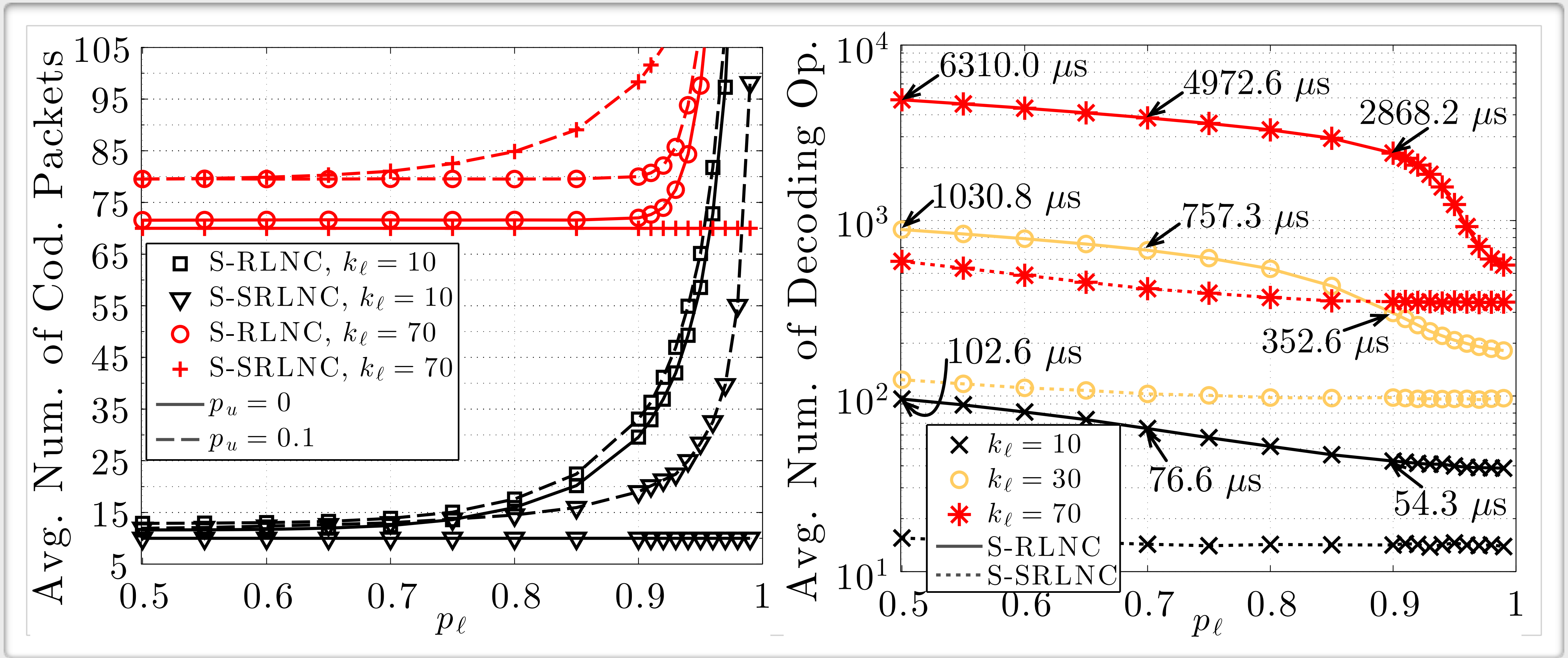
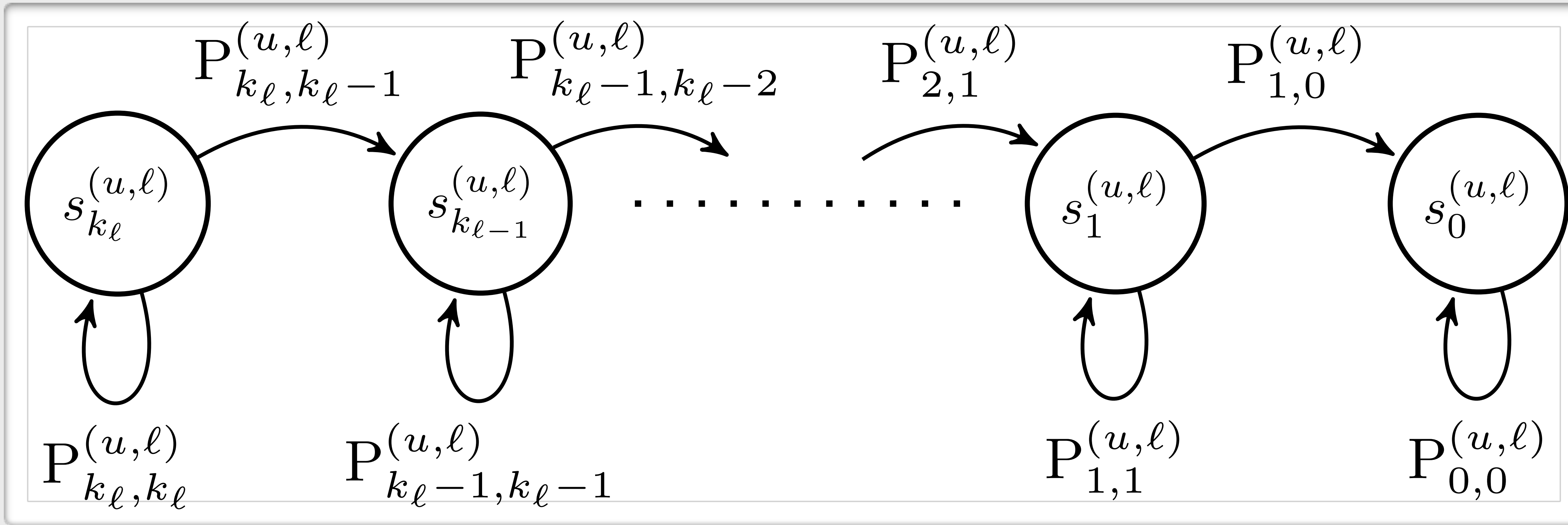
Should I Stay or Should I Go?



- Some actual performance value obtained by implementing a sparse RNC decoder in a Raspberry Pi Model B.
- If you can afford to wait, than sparsity is your best fit!

Markovian Model

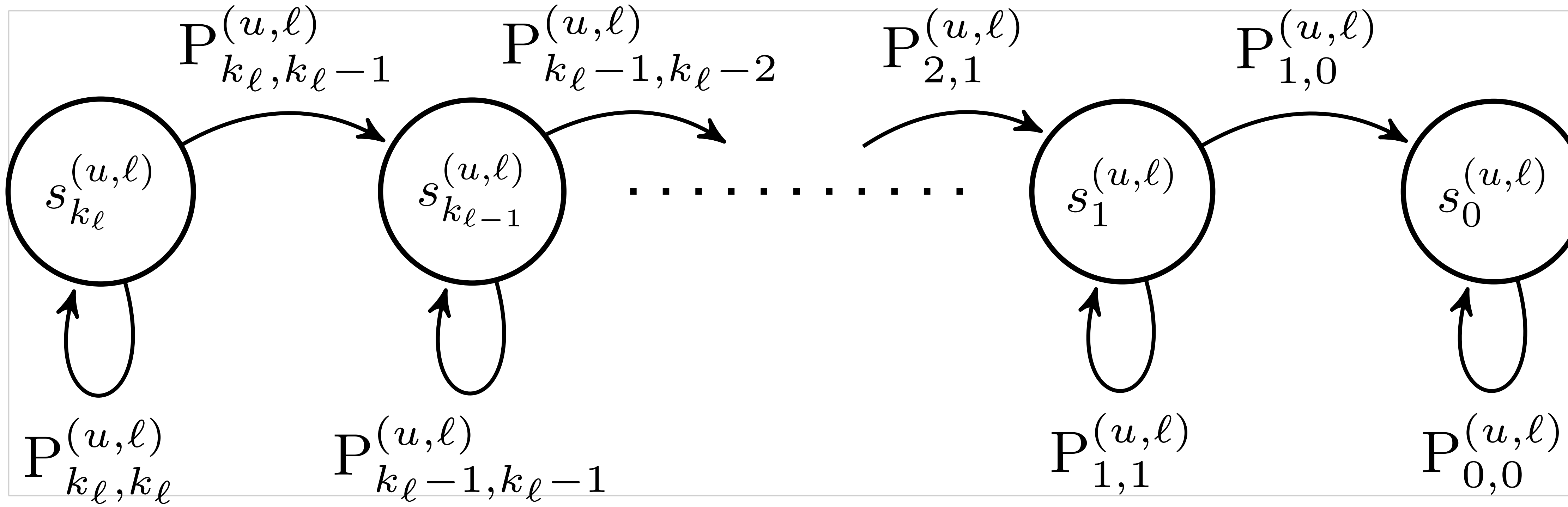
- Our chain models the process (seen from the perspective of the receiver) of building a full-rank coding matrix $\mathbf{C}_u$



- The chain is in state $s^{(u,\ell)}_{k_\ell}$ if k_ℓ linearly independent coded packets are still missing.

- That is an Absorbing Markov Chain (AMC)!

Markovian Model



Unknown! We used an upper-bound!

User PER for a given MCS m

$$P^{(u,\ell)}_{i,j} = \begin{cases} (1 - P_{\ell,k_\ell-i})[1 - p_u(m)] & \text{if } i - j = 1 \\ P_{\ell,k_\ell-i}[1 - p_u(m)] + p_u(m) & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

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DoF Probability

- Assume that $\mathbf{C}_u$ consists of $(t+1) \times k_\ell$ elements. Assume that t out of t+1 rows are linearly independent.

- $\mathrm{P}_{\ell,t}$ is the probability that $\mathbf{C}_u$ is not full-rank and is upper-bounded as follows*:

$$\mathrm{P}_{\ell,t} \leq \left[\max\left(p_\ell, \frac{1-p_\ell}{q-1}\right)\right]^{k_\ell - t}$$

- That bound is exact for $p_\ell = 1/q$, we have

$$\mathrm{P}_{\ell,t} = \frac{q^t}{q^{k_\ell}}$$

---

* J. Blömer, R. Karp, and E. Welzl, "The Rank of Sparse Random Matrices Over Finite Fields," *Random Structures & Algorithms*, vol. 10, no. 4, pp. 407–419, 1997.

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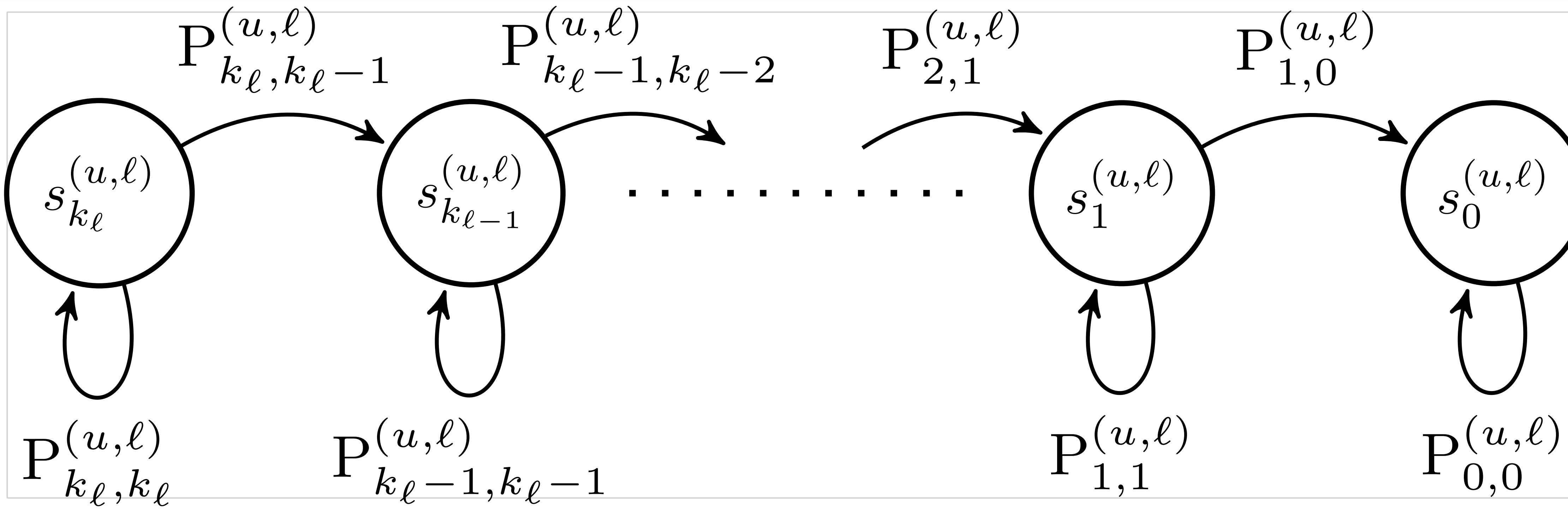
Markovian Model



$$\mathbf{T}^{(u,\ell)} \doteq \left[\begin{array}{c|c} 1 & \mathbf{0} \\ \hline \mathbf{R}^{(u,\ell)} & \mathbf{Q}^{(u,\ell)} \end{array}\right]$$

AMC transition matrix

Fundamental matrix

$$\mathbf{N}^{(u,\ell)} = \sum_{t=0}^{\infty} \left(\mathbf{Q}^{(u,\ell)}\right)^t = \left[\mathbf{I} - \mathbf{Q}^{(u,\ell)}\right]^{-1}$$

Markovian Model

- Why is the fundamental matrix so important? Its (i,j) element is the avg. number of coded packet transmissions needed (to a system started at the i-th state) to get to the j-th state.

- Hence, the avg number of transmissions needed to get to the absorbing state (for a system started in the i-th state) is

$$\tau_i^{(u,\ell)} = \begin{cases} 0 & \text{if } i = 0 \\ \sum_{j=1}^{i} \mathrm{N}_{i,j}^{(u,\ell)} & \text{if } i = 1, \ldots, k_\ell. \end{cases}$$

- In the non-systematic RNC we have

$$\tau_{\text{S-RLNC}}^{(u,\ell)} = \tau_{k_\ell}^{(u,\ell)}$$

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3. Resource Allocation for Sparse RLNC

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Proposed RA Model

- The objective of the problem is to minimize the computational complexity of the decoding operations.

- The main constraints are those which impose an upper-limit to the avg. number of transmissions needed to allow a target number of users to recover a certain layer.

> The solution is not trivial… but we derived an analytical solution via referring to tools belonging to the convex optimization domain. *The solution is safe!*

$$
\begin{aligned}
\text{ST} \quad & \max_{\substack{p_1,\ldots,p_L\\ m_1,\ldots,m_L}} \quad \|\mathbf{p}\|_1 \\
\text{s.t.} \quad & \sum_{u=1}^{U} \delta\left(\tau_{\text{S-RLNC}}^{(u,\ell)} \leq \hat{\tau}_\ell\right) \geq \hat{U}_\ell, && \ell = 1,\ldots,L \\
& q^{-1} \leq p_\ell < 1 && \ell = 1,\ldots,L \\
& m_\ell \in \{1,\ldots,M\} && \ell = 1,\ldots,L
\end{aligned}
$$

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Proposed RA Model

$$
\begin{aligned}
\text{ST} \quad & \boxed{\max_{\substack{p_1,\ldots,p_L \\ m_1,\ldots,m_L}} \quad \|\mathbf{p}\|_1} \\
\text{s.t.} \quad & \boxed{\sum_{u=1}^{U} \delta\left(\tau_{\text{S-RLNC}}^{(u,\ell)} \leq \hat{\tau}_\ell\right) \geq \hat{U}_\ell, \qquad \ell = 1,\ldots,L} \\
& \boxed{q^{-1} \leq p_\ell < 1 \qquad \ell = 1,\ldots,L} \\
& m_\ell \in \{1,\ldots,M\} \qquad \ell = 1,\ldots,L
\end{aligned}
$$

- The sum of the sparsity of each layer is maximized
- The no. of UEs experiencing at most the target avg. recovery delay shall be greater than or equal to a predetermined fraction
- RNC shall go sparse and not dense!

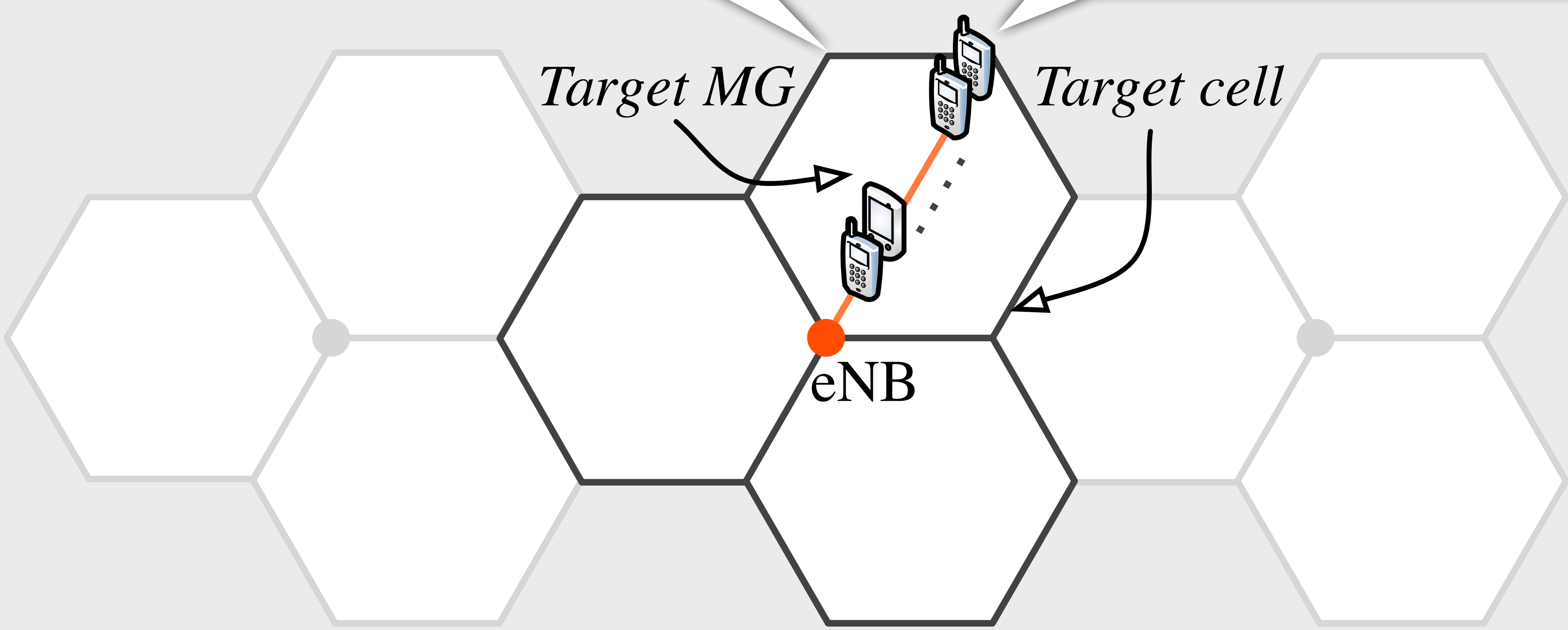
4. Numerical Results

Simulated Scenario

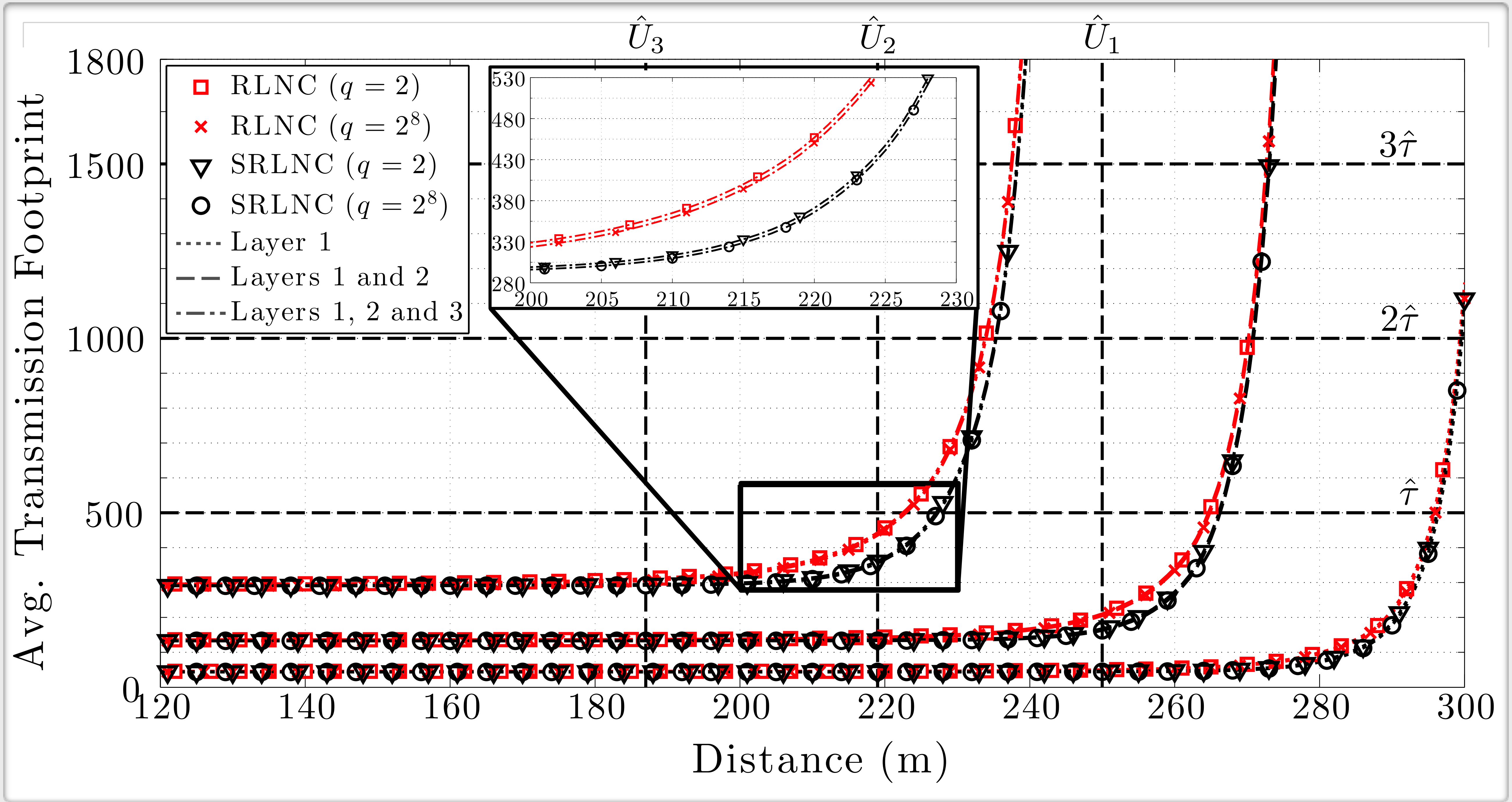
- LTE-A eMBMS scenarios

Scenario with a high heterogeneity. 80 UEs equally spaced and placed along the radial line representing the symmetry axis of one sector of the target cell

We considered Stream A and B which have 3 layers, bitrate of A is smaller than that of B



Stream A



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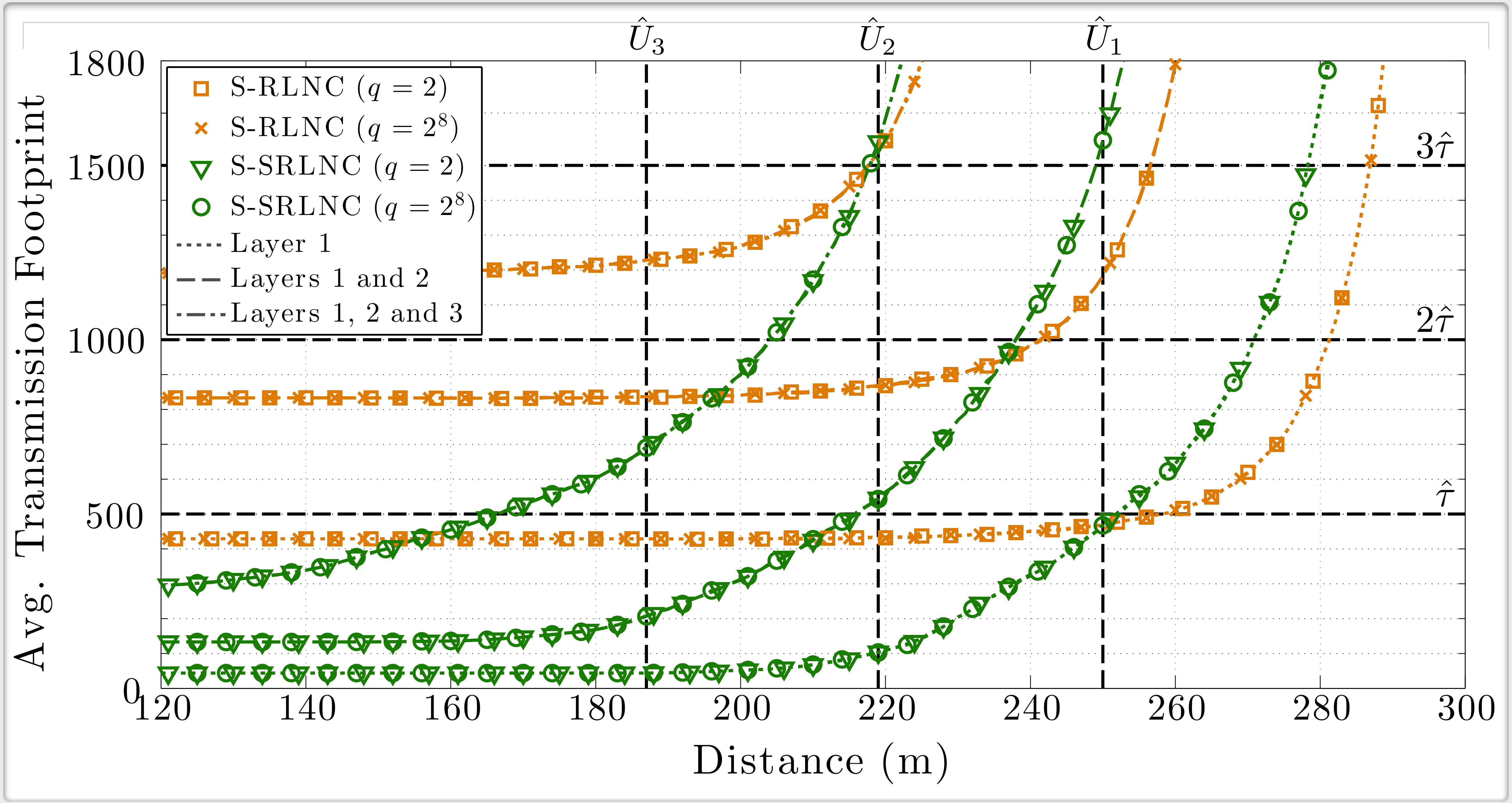
Stream A



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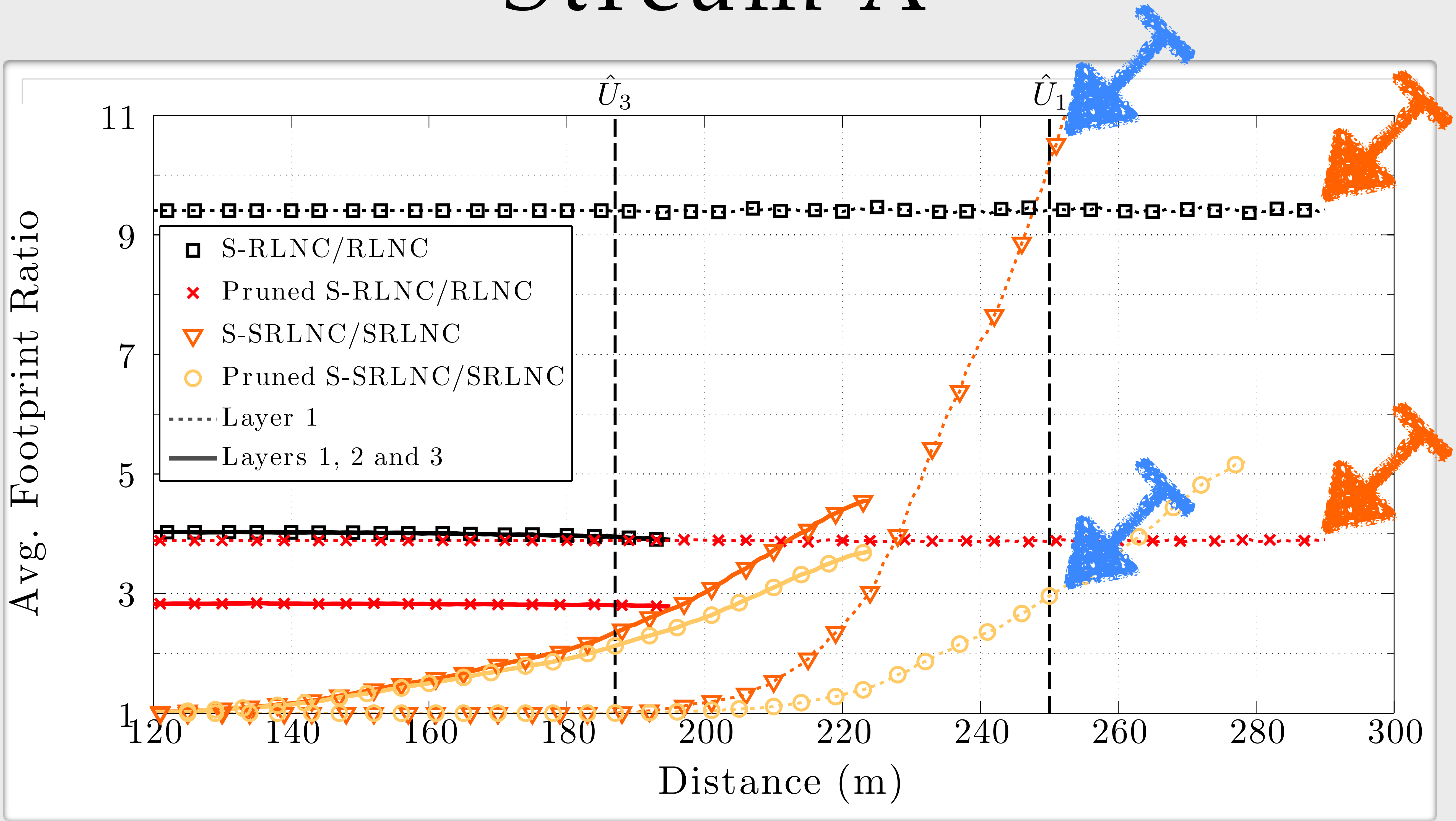
Stream A



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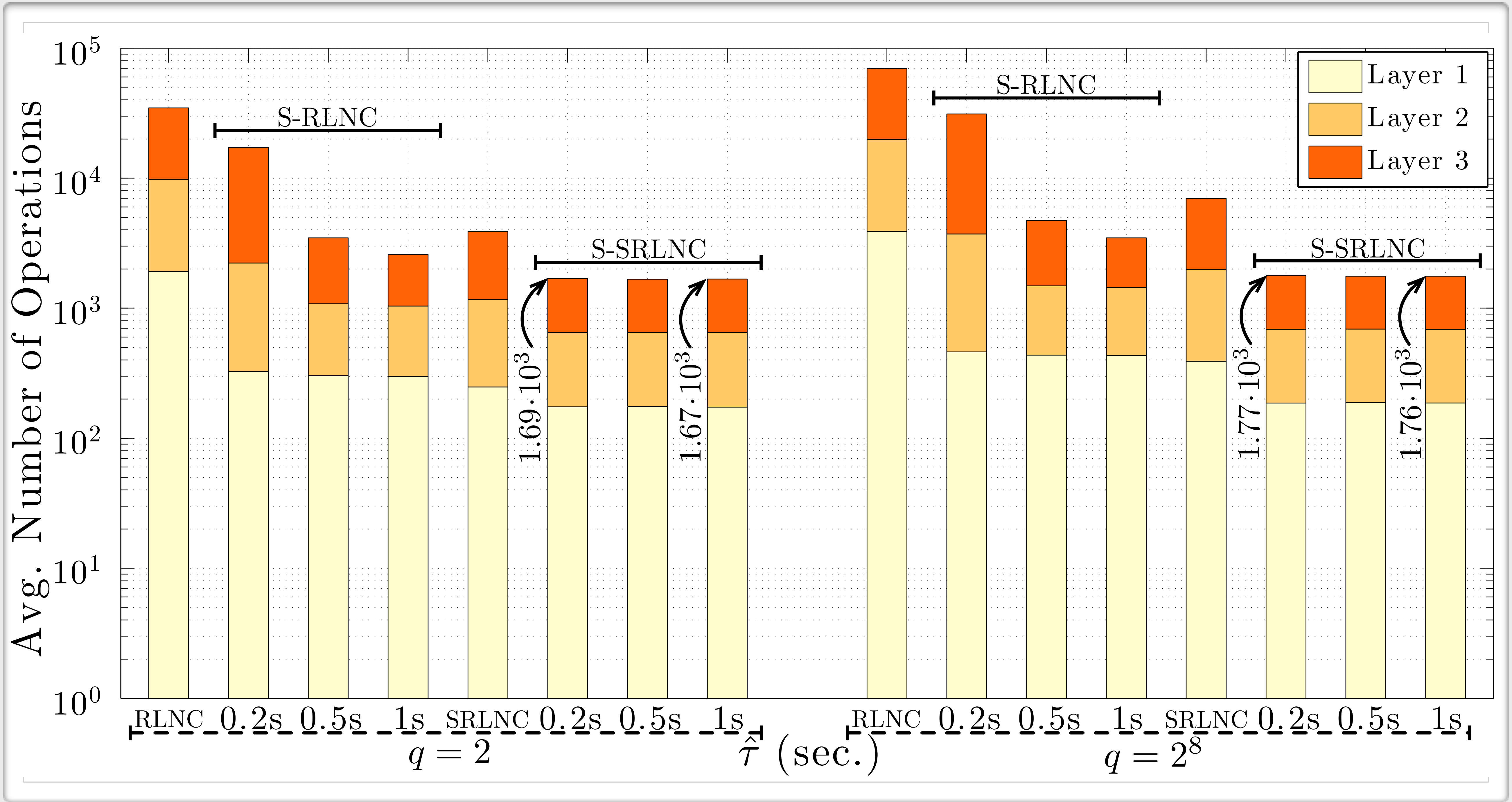
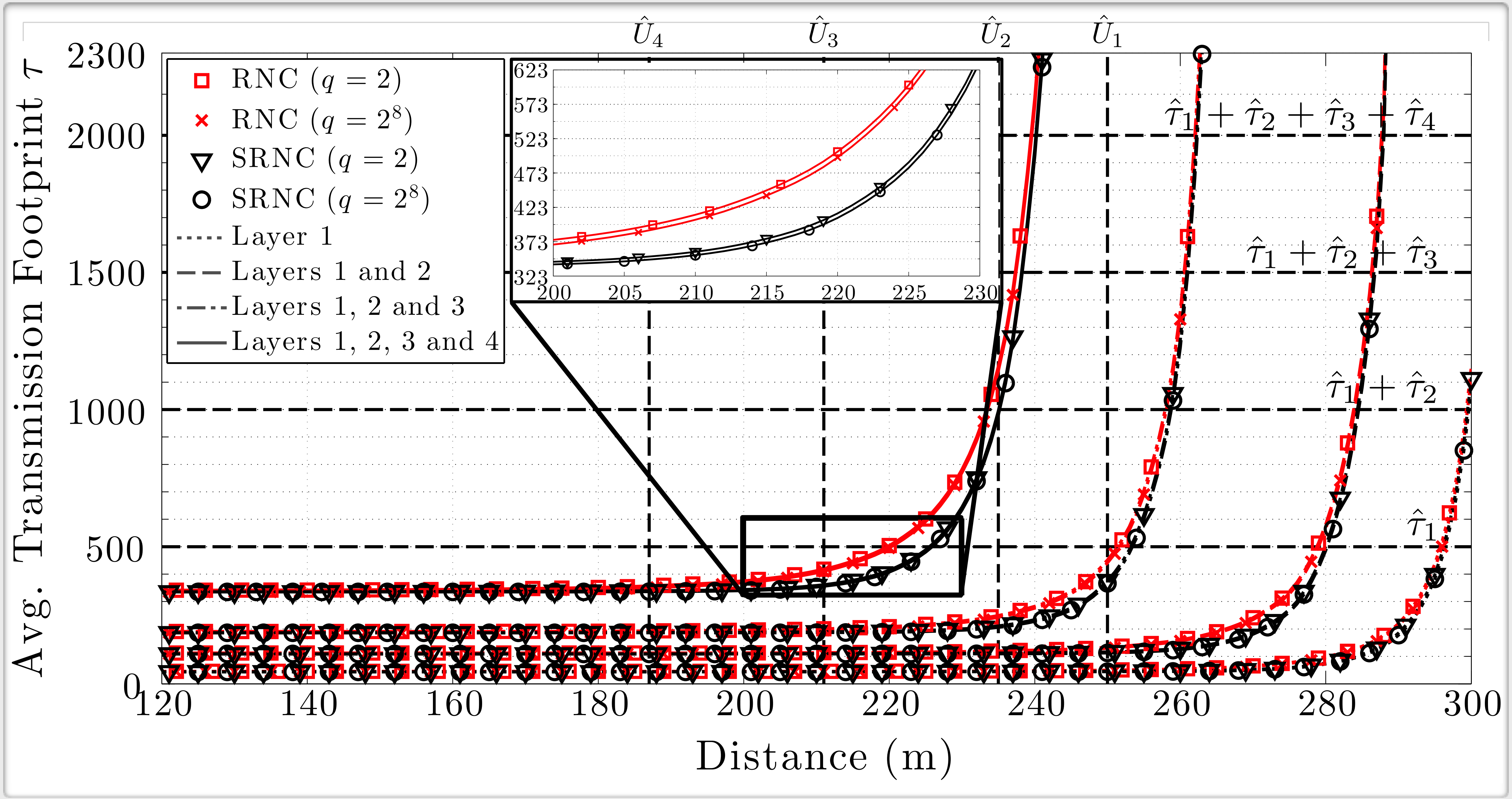
Stream A



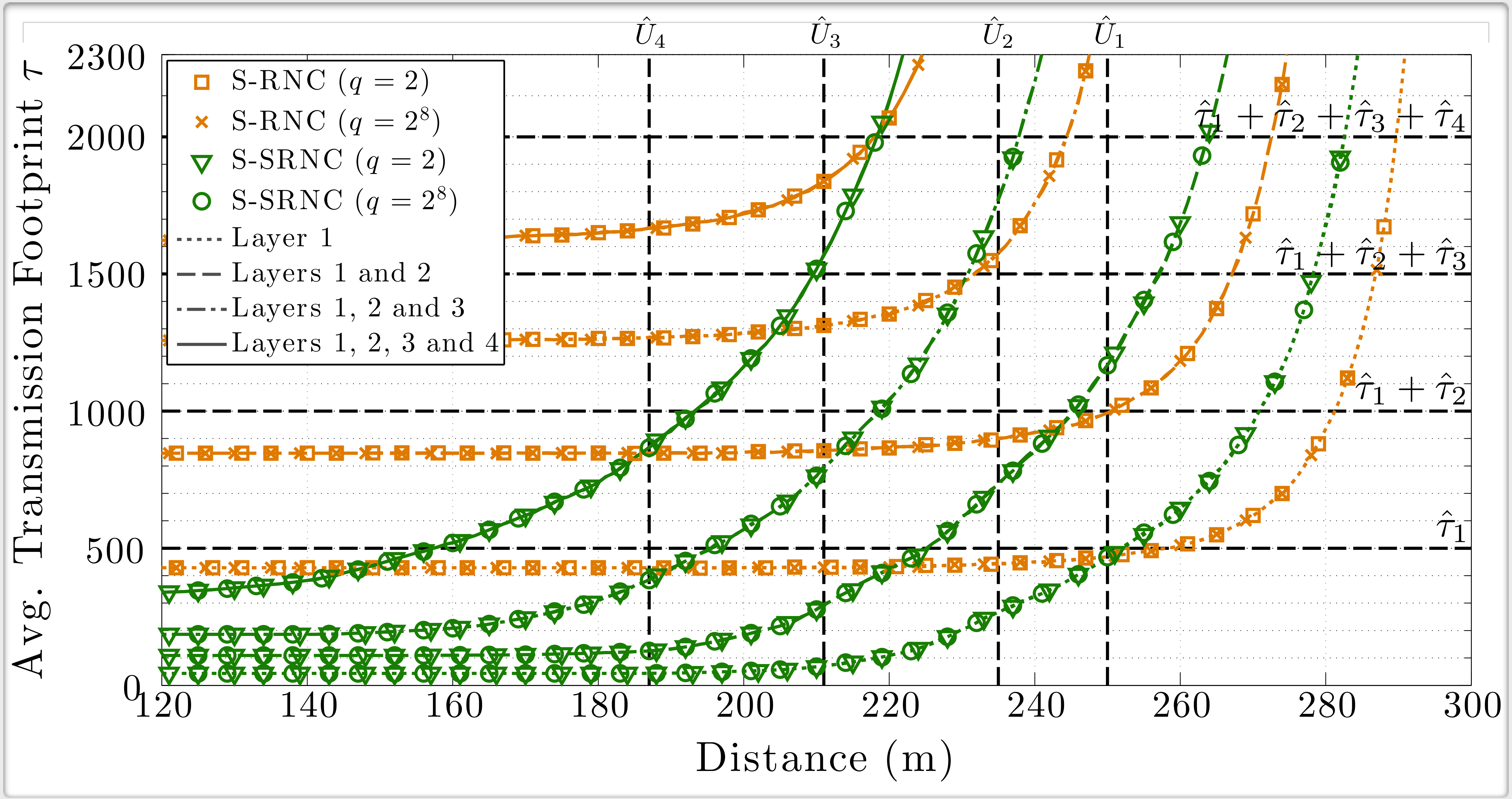
Avg. Number of Operations
10^0
10^1
10^2
10^3
10^4
10^5
S-RLNC
S-SRLNC
S-RLNC
S-SRLNC
$1.69 \cdot 10^3$
$1.67 \cdot 10^3$
$1.77 \cdot 10^3$
$1.76 \cdot 10^3$
Layer 1
Layer 2
Layer 3
RLNC 0.2s 0.5s 1s SRLNC 0.2s 0.5s 1s
RLNC 0.2s 0.5s 1s SRLNC 0.2s 0.5s 1s
$q = 2$
$\hat{\tau}$ (sec.)
$q = 2^8$

Stream B



RNC ($q = 2$)
RNC ($q = 2^8$)
SRNC ($q = 2$)
SRNC ($q = 2^8$)
Layer 1
Layers 1 and 2
Layers 1, 2 and 3
Layers 1, 2, 3 and 4
$\hat{U}_4$
$\hat{U}_3$
$\hat{U}_2$
$\hat{U}_1$
$\hat{\tau}_1 + \hat{\tau}_2 + \hat{\tau}_3 + \hat{\tau}_4$
$\hat{\tau}_1 + \hat{\tau}_2 + \hat{\tau}_3$
$\hat{\tau}_1 + \hat{\tau}_2$
$\hat{\tau}_1$
Avg. Transmission Footprint τ
Distance (m)
0
500
1000
1500
2000
2300
120
140
160
180
200
220
240
260
280
300
323
373
423
473
523
573
623
200
205
210
215
220
225
230

Stream B



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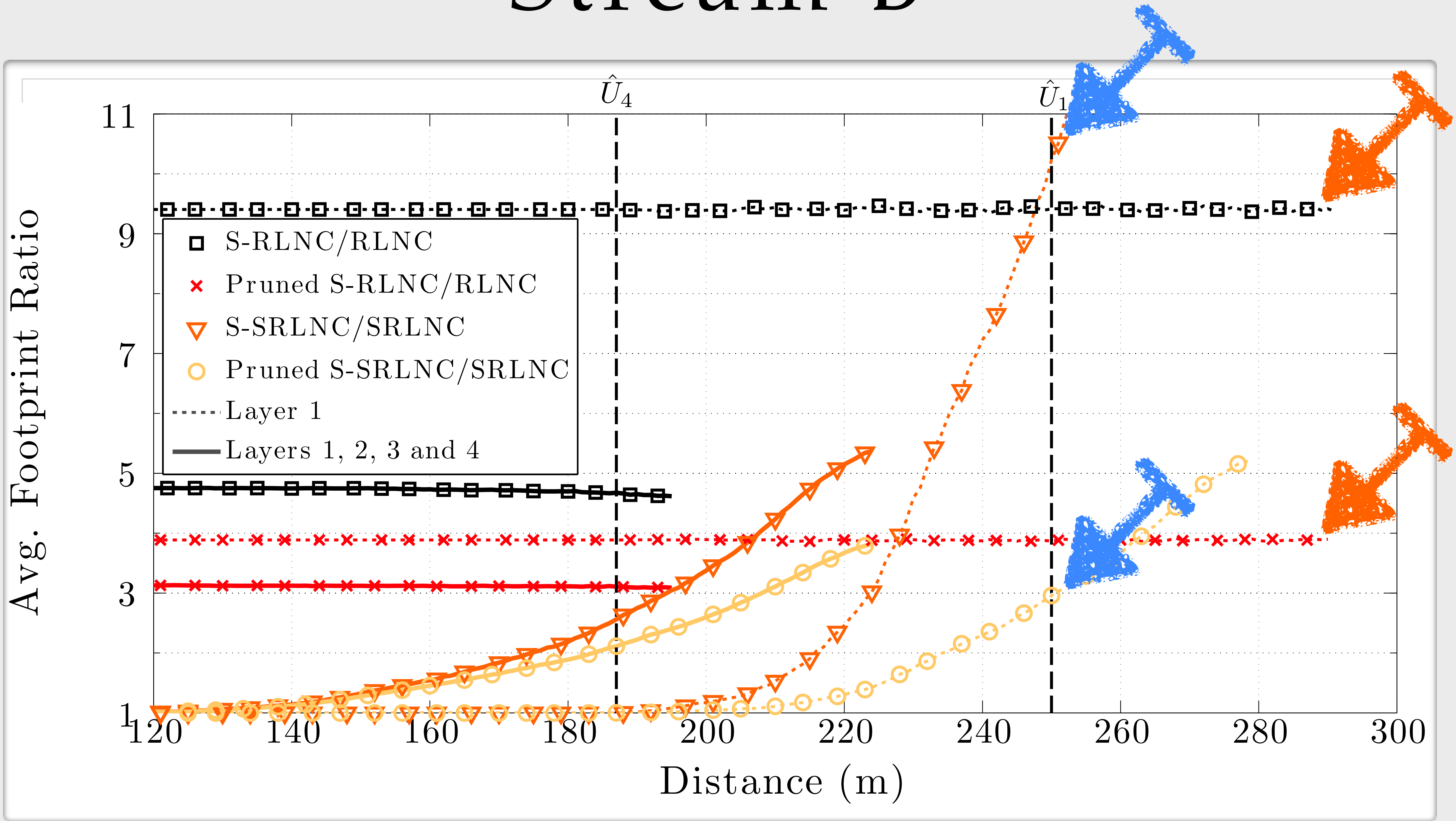
Stream B



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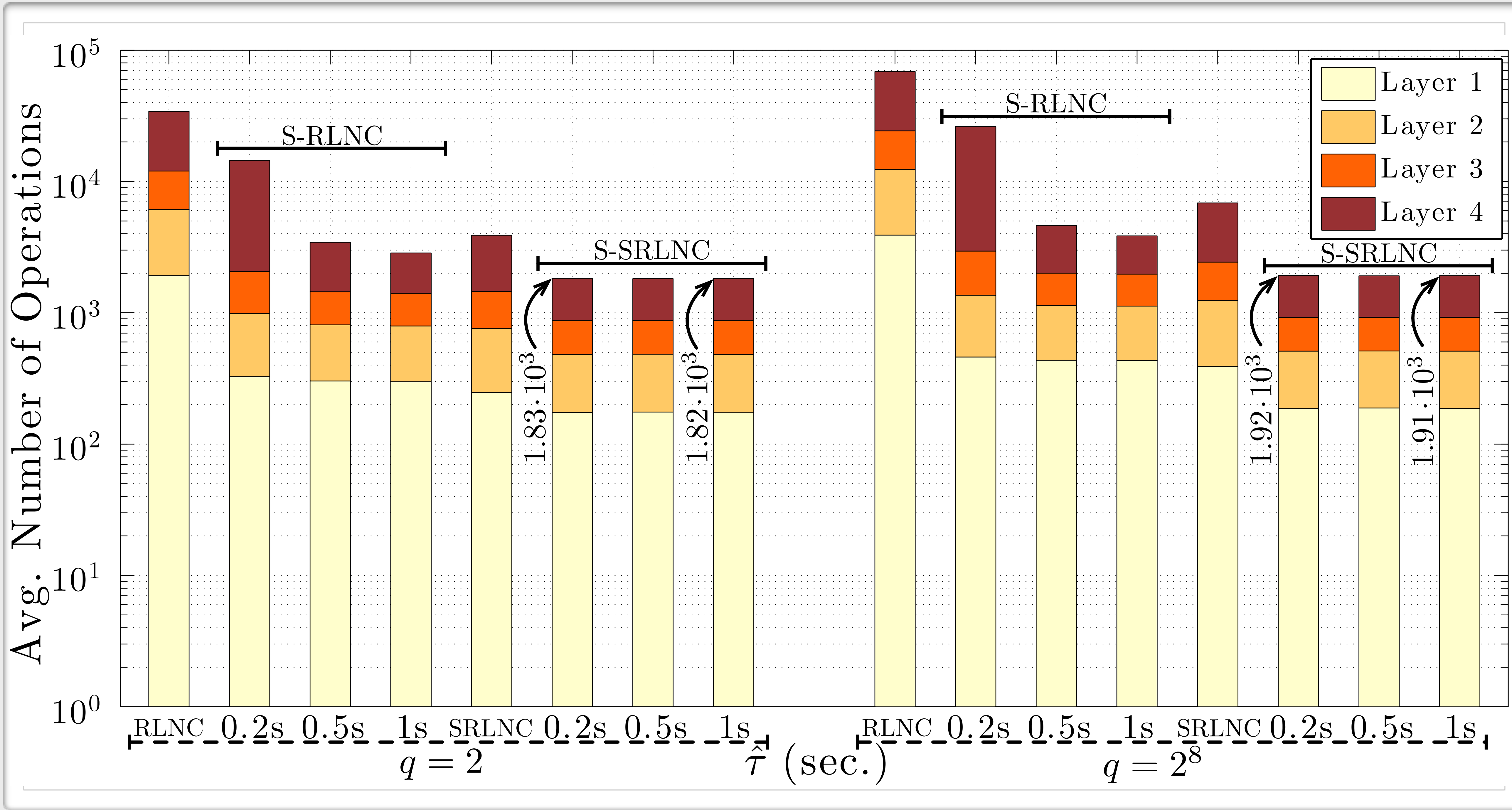
Stream B



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5. Concluding Remarks

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Concluding Remarks

- We addressed the issue of the complexity of a generic network coding decoder, in a multicast network scenario.
- We proposed a constrained convex resource allocation framework suitable for jointly optimizing both the MCS indexes and the code sparsity.
- The objective of the optimization model is that of minimizing the number of operations performed by a generic network coding decoder employing GE.
- The average transmission footprint is likely to increase as the sparsity of the code grows. However, this drawback is greatly reduced by simply avoiding the transmissions of all-zero coding vectors.
- The proposed optimization ensures a reduction in the average number of decoding operations of at least 57%.

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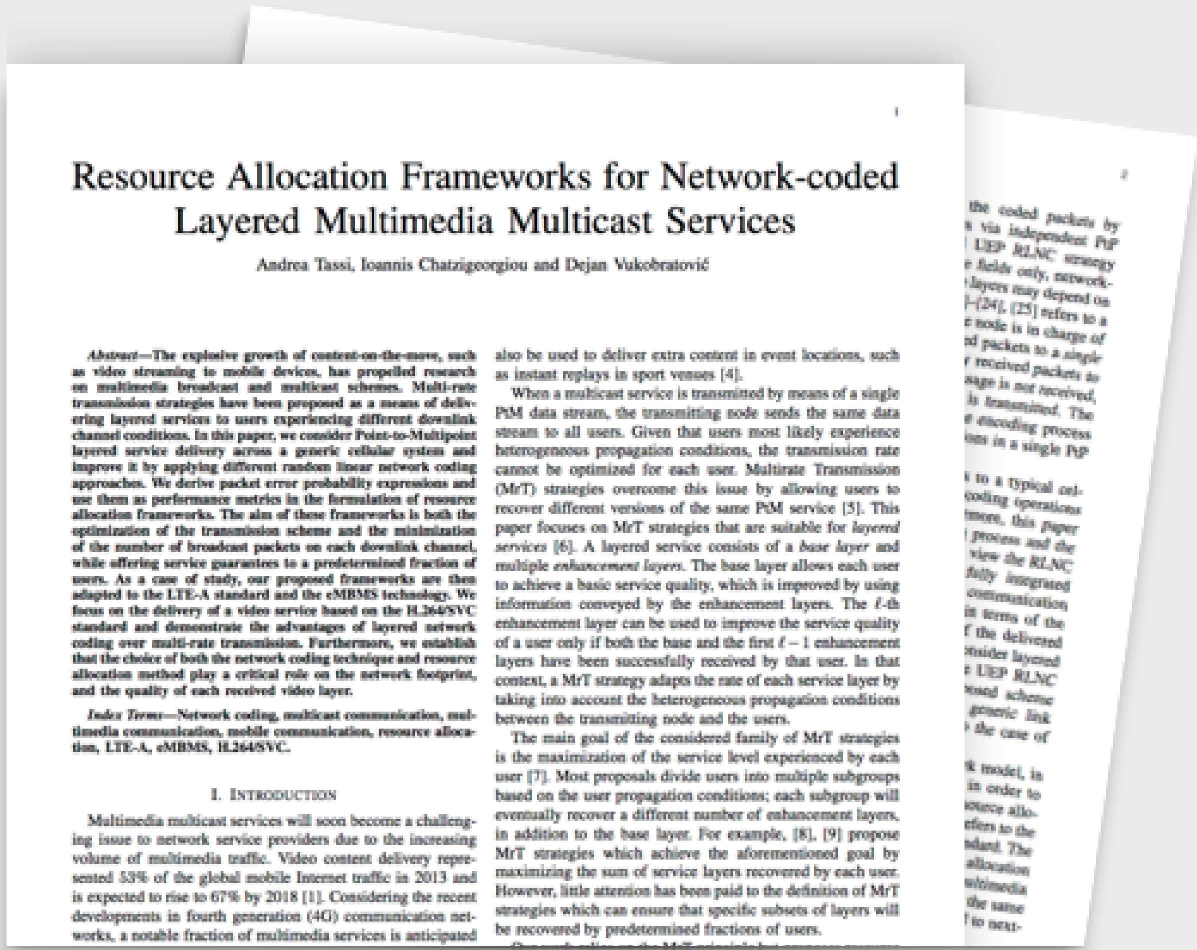
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Resource Allocation Frameworks for Network-coded Layered Multimedia Multicast Services

Andrea Tassi, Ioannis Chatzigeorgiou and Dejan Vukobratović

Abstract—The explosive growth of content-on-the-move, such as video streaming to mobile devices, has propelled research on multimedia broadcast and multicast schemes. Multi-rate transmission strategies have been proposed as a means of delivering layered services to users experiencing different downlink channel conditions. In this paper, we consider Point-to-Multipoint layered service delivery across a generic cellular system and improve it by applying different random linear network coding approaches. We derive packet error probability expressions and use them as performance metrics in the formulation of resource allocation frameworks. The aim of these frameworks is both the optimization of the transmission scheme and the minimization of the number of broadcast packets on each downlink channel, while offering service guarantees to a predetermined fraction of users. As a case of study, our proposed frameworks are then adapted to the LTE-A standard and the eMBMS technology. We focus on the delivery of a video service based on the H.264/SVC standard and demonstrate the advantages of layered network coding over multi-rate transmission. Furthermore, we establish that the choice of both the network coding technique and resource allocation method play a critical role on the network footprint, and the quality of each received video layer.

Index Terms—Network coding, multicast communication, multimedia communication, mobile communication, resource allocation, LTE-A, eMBMS, H.264/SVC.

I. INTRODUCTION

Multimedia multicast services will soon become a challenging issue to network service providers due to the increasing volume of multimedia traffic. Video content delivery represented 53% of the global mobile Internet traffic in 2013 and is expected to rise to 67% by 2018 [1]. Considering the recent developments in fourth generation (4G) communication networks, a notable fraction of multimedia services is anticipated … also be used to deliver extra content in event locations, such as instant replays in sport venues [4].

When a multicast service is transmitted by means of a single PtM data stream, the transmitting node sends the same data stream to all users. Given that users most likely experience heterogeneous propagation conditions, the transmission rate cannot be optimized for each user. Multirate Transmission (MrT) strategies overcome this issue by allowing users to recover different versions of the same PtM service [5]. This paper focuses on MrT strategies that are suitable for *layered services* [6]. A layered service consists of a *base layer* and multiple *enhancement layers*. The base layer allows each user to achieve a basic service quality, which is improved by using information conveyed by the enhancement layers. The ℓ-th enhancement layer can be used to improve the service quality of a user only if both the base and the first $\ell - 1$ enhancement layers have been successfully received by that user. In that context, a MrT strategy adapts the rate of each service layer by taking into account the heterogeneous propagation conditions between the transmitting node and the users.

The main goal of the considered family of MrT strategies is the maximization of the service level experienced by each user [7]. Most proposals divide users into multiple subgroups based on the user propagation conditions; each subgroup will eventually recover a different number of enhancement layers, in addition to the base layer. For example, [8], [9] propose MrT strategies which achieve the aforementioned goal by maximizing the sum of service layers recovered by each user. However, little attention has been paid to the definition of MrT strategies which can ensure that specific subsets of layers will be recovered by predetermined fractions of users.

For more information

http://arxiv.org/abs/1411.5547

or

http://goo.gl/Z4Y9YF

A. Tassi, I. Chatzigeorgiou, and D. Vukobratović, "**Resource Allocation Frameworks for Network-coded Layered Multimedia Multicast Services**",

IEEE Journal on Selected Areas in Communications, Special Issue on "Fundamental Approaches to Network Coding in Wireless Communication Systems", *in press*.

Thank you for your attention

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